An Analysis of Random Search

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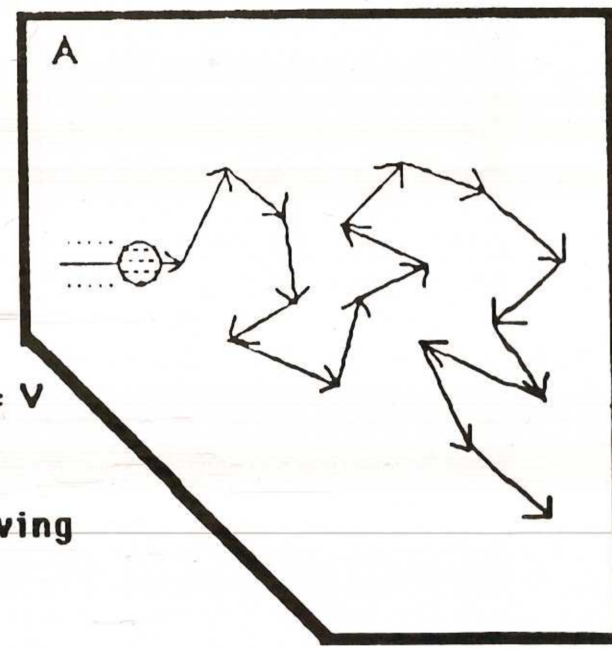
Koopman's Random Search Formula

assumes: OEG #56

1 -- target's position is uniformly distributed in area

2 -- observor's path is random in area: portions not too near are placed independently

3 -- cookie cutter detector: range R



$$P_D(t) = 1 - \left(1 - \frac{2RVt}{nA}\right)^n$$

$$\longrightarrow 1 - e^{-2RVt/A}$$

- target stationary, searcher velocity = V (or vice versa)
- 'dynamic enhancement' if both are moving

Problems with this Model : & QUESTIONS

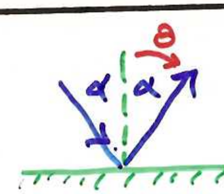
- how can a continuous track have independent 'segments'?
- are there any searcher/target motion models consistent
  a) with the formula? b) with uniform density?
  - diffusion? ← random tour? → constant velocity?
- what course change occurs at boundaries of the area
  - specular reflection? - diffuse reflection? - uniform angle?
- is the formula valid for all 't' ?
- ✓ are there finite area correction factors (i.e. $\frac{\pi R^2}{A} \neq 0$)
- is there a more general formulation of 'random search' that
  -- we can understand ?
  -- has a wider range of validity ?

Simulation Results

- Cases:
  - <u>target motion</u>
    - diffusion
    - random tour
    - constant velocity

> λ = rate of course changes
>
> V = velocity of target
> (searcher stationary)
>
> diffusion coefficient --> $D = \frac{V^2}{\lambda}$

  - <u>boundaries</u>
    - specular reflection



$\theta = \alpha$

    - diffuse reflection   pdf $= \frac{1}{2}\cos\theta$
    - uniform reflection angle $= \frac{1}{\pi}$ ← N. G.

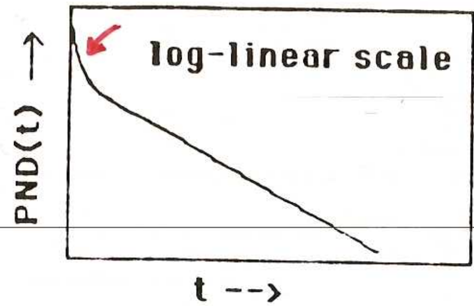
- Least squares curve fit: $P_D(t) = 1 - \left(1 - \frac{\pi R^2}{A}\right) e^{-\gamma t}$

$$\gamma = \frac{24.7\, R V^2}{(A - \pi R^2)^{1.5} \lambda} \left[1 - e^{-.084 \frac{\lambda \sqrt{A}}{V}}\right]$$

constant velocity limit ($\lambda \to 0$)	-- diffusion limit ($\lambda\sqrt{A}/V \to \infty$) ← $\ni \frac{V^2}{\lambda} = \text{const}$
$\gamma \dashrightarrow \frac{2.075\, R V \sqrt{A}}{(A - \pi R^2)^{1.5}}$	$\gamma \dashrightarrow \frac{24.7\, R V^2}{(A - \pi R^2)^{1.5}\lambda} = \frac{24.7\, R\, D}{(A - \pi R^2)^{1.5}}$

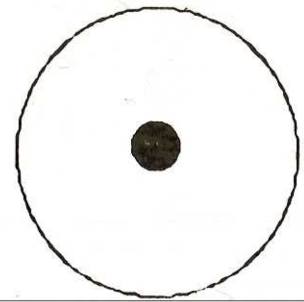
Explaining the Simulation Results

- diffusion limit:



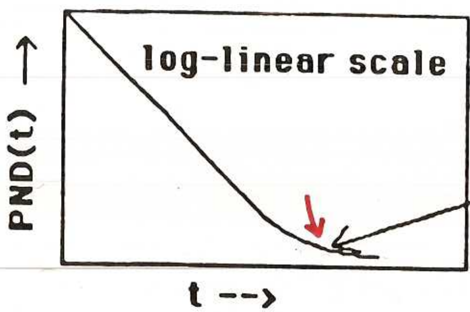
Fokker-Planck eq.
(heat flow)

only exponential
asymptotically

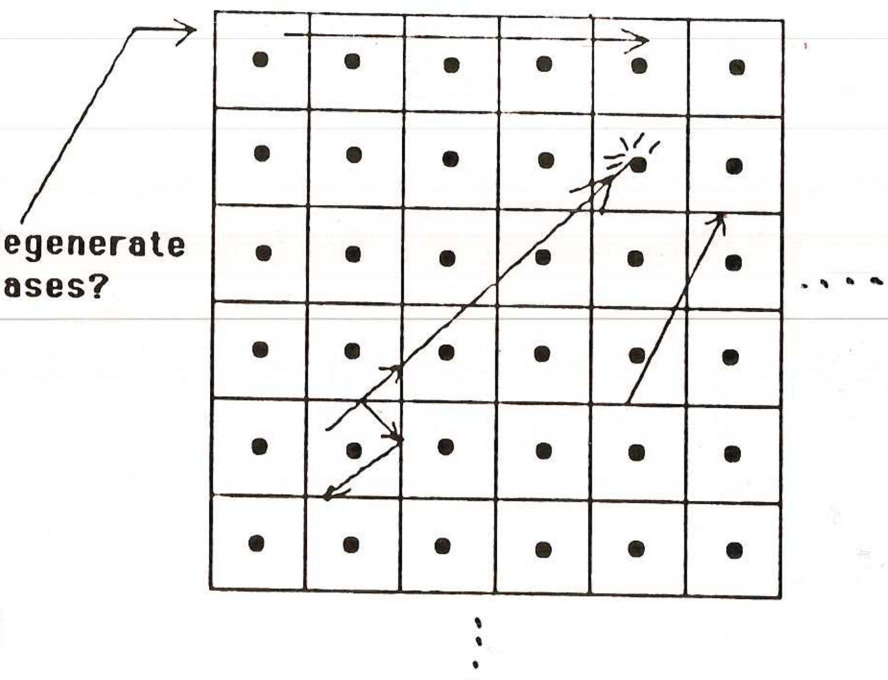


$$\sum_n^{\infty} e^{-\lambda_n t} \phi_n(\vec{x})$$

- Why does opposite limit, V = const., => Random search?



degenerate
cases?



-- method of images
for specular reflection

-- small amount of noise =>
"similar" to Poisson field

Minimal, Realistic Assumptions

- one target, one searcher in finite area A
- approximately constant velocity until reach boundary
- specular reflection at boundary (sufficient, not necessary)
- either target or searcher can be thought of as moving
- require "small" noise added to velocity, i.e. small change over distance of $\sqrt{A}$
- 'mover' is uniformly distributed over A, uniform bearing pdf

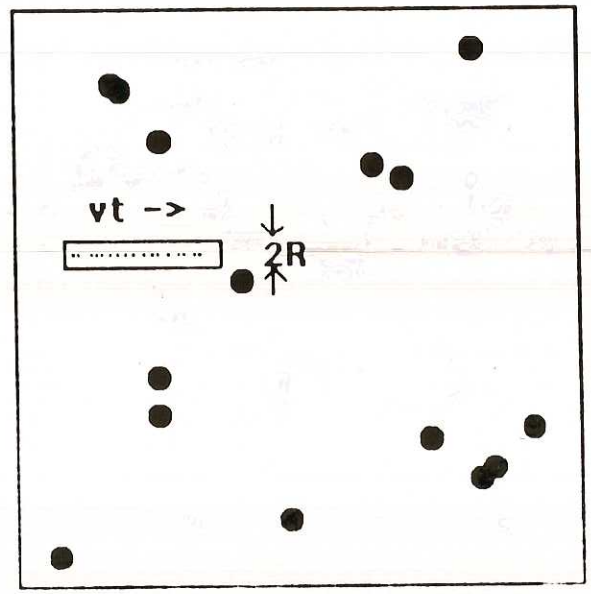
Poisson Field Model

- density of targets, $\rho = \dfrac{1 \text{ target}}{(A - \pi R^2)\ NM^2}$

– PND(t) =

P_r {0 dots in swept area}

$= e^{-2RVt\rho}$ Poisson



- except for density, exactly Koopman's formula

$\gamma = \dfrac{2RV}{A - \pi R^2}$

- but, there is wasted effort,

  -- overlap of 'detectors'
  -- shadowing of some close together detectors (or targets)

- Hence, random search in the lattice (or specular reflection) must be more efficent than search in Poisson field

=> Koopman's formula underestimates the detection rate.

Compare with 'Empirical' Equation

- Consider detection rate as $\lambda \dashrightarrow 0$ (from Simulation)

$$\gamma \dashrightarrow \frac{2.075\, R\, V \sqrt{A}}{(A - \pi R^2)^{1.5}} \approx \frac{2.0 \quad R\, V}{(A - \pi R^2)} \left[\frac{1}{\left(1 - \frac{\pi R^2}{A}\right)^{.5}} \right].$$

only 2.075 → 2.0

- Hence, the simulation results yield a greater detection rate for finite search area, greater by the factor $\frac{1}{\left(1 - \frac{\pi R^2}{A}\right)^{.5}}$ than Koopman/Poisson Field

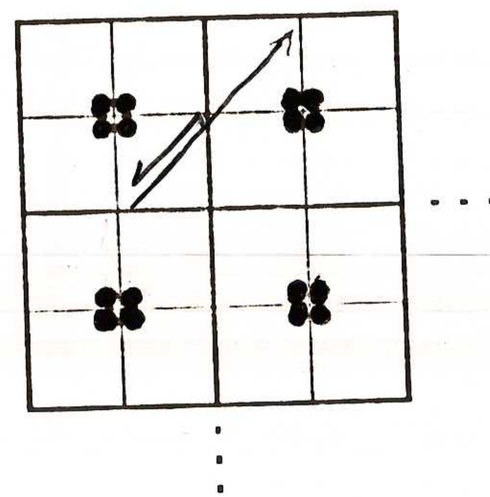
- Question, our qualitative arguments re shadowing in a Poisson field explain the greater detection rate, qualitatively; can it explain it quantitatively?

Edge effects and Shadowing

- Q. will position of stationary detector in

  area influence detection rate?

- Analyze with images



- A --> 4 A

  R --> $\gtrsim$ 2R $\Rightarrow \dfrac{\pi R^2}{A} \;\text{-->}\; \dfrac{\pi R^2}{A}$

  $4R \le W \le 4.8R$

  (since no overlapping of detectors as for Poisson field)

  -BUT, clearly shadowing is significant

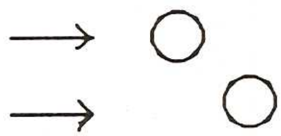
  Effective sweep width of the 'large detector' is $4R+\epsilon$

  (NOT $4*2R$)

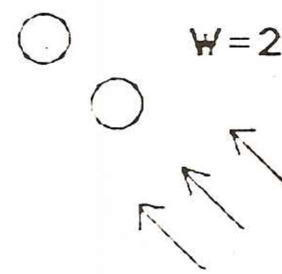
- detection rate γ --> $\approx 1/2\,\gamma$

A Theory of "Shadowing"

- Consider two detectors r NM apart
  - the sweep width will vary between 2R and 2*(2R) depending on the direction of motion (relative)



W = 4R

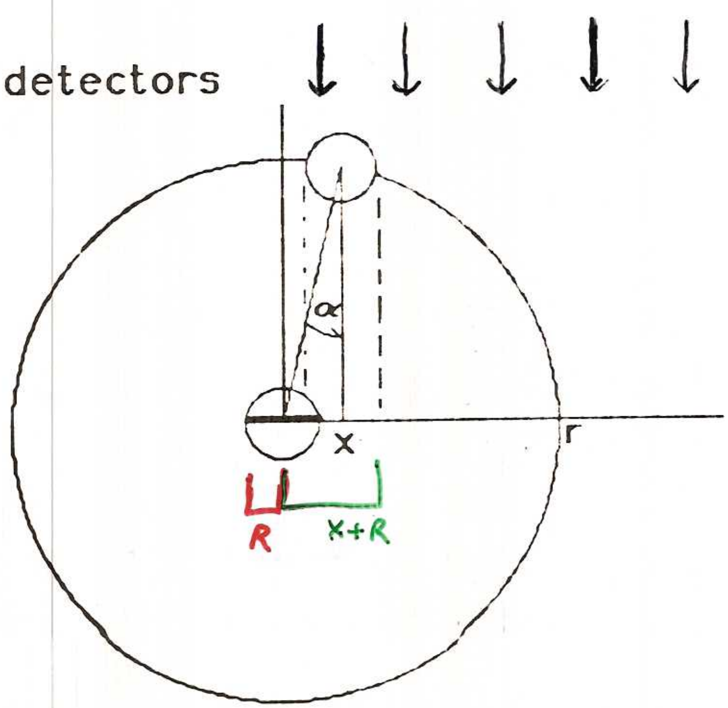


W=2R

-- if r >> R the effective sweep width is 2*(2R)

-- if r $\gtrsim$ R then W will be a r.v., depending on the direction of target or detector motion

- Shadowing of 2 detectors



- x>2R=> W=4R

- x<2R =>

  W = R+(x+R)

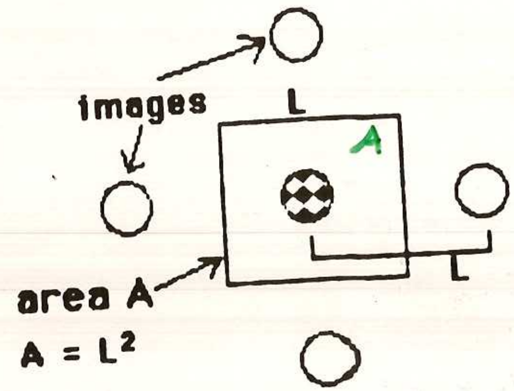
for r > 2R,

E{W} =

$$4R + \frac{2r}{\pi}\left(1-\sqrt{1-\frac{4R^2}{r^2}}\right) - \frac{4R}{\pi}\arcsin\left(\frac{2R}{r}\right) \xrightarrow[r\to\infty]{} 4R$$

Shadowing of Lattice vs. Poisson Field of Detectors

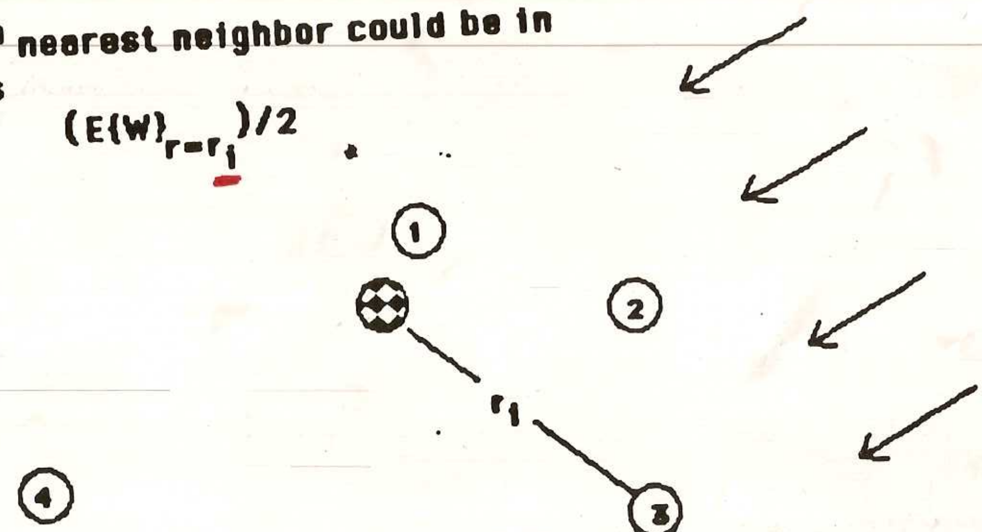
- Consider only 4 nearest neighbors



For Lattice:

- Effective sweep width after shadowing by one of nearest neighbors is $(E\{W\}_{r=L})/2$
  (relative to infinite area, no boundaries)
  - use $r = L$ for each of the 4 nearest neighbors

- For Poisson field, i^{th} nearest neighbor could be in any of four quadrants
  $(E\{W\}_{r=r_i})/2$



Average over a Poisson Field

- distribution of n^{th} nearest neighbors (pdf)

$$f_n(r)dr = P_r[n-1 \text{ in area } \pi r^2] \cdot P_r[1 \text{ in } 2\pi r\, dr]$$

$$= \frac{(\rho\pi r^2)^{n-1}}{(n-1)!} e^{-\rho\pi r^2} * (\rho\, 2\pi r\, dr)$$

- $E[R_n] = \dfrac{(2n-1)!!}{(n-1)!\, 2^n \sqrt{\rho}}$, $E[R_n^{-1}] = \dfrac{(2n-3)!!\, 2\pi}{(n-1)!\, 2^n \sqrt{\rho}}$

-

n	1	2	3	4
$E[R_n]$	.5 L	.75 L	.94 L	1.09 L
$E[R_n^{-1}]$	3.14/L	1.57/L	1.18/L	.98/L

Compensated Detection Rate - Lattice

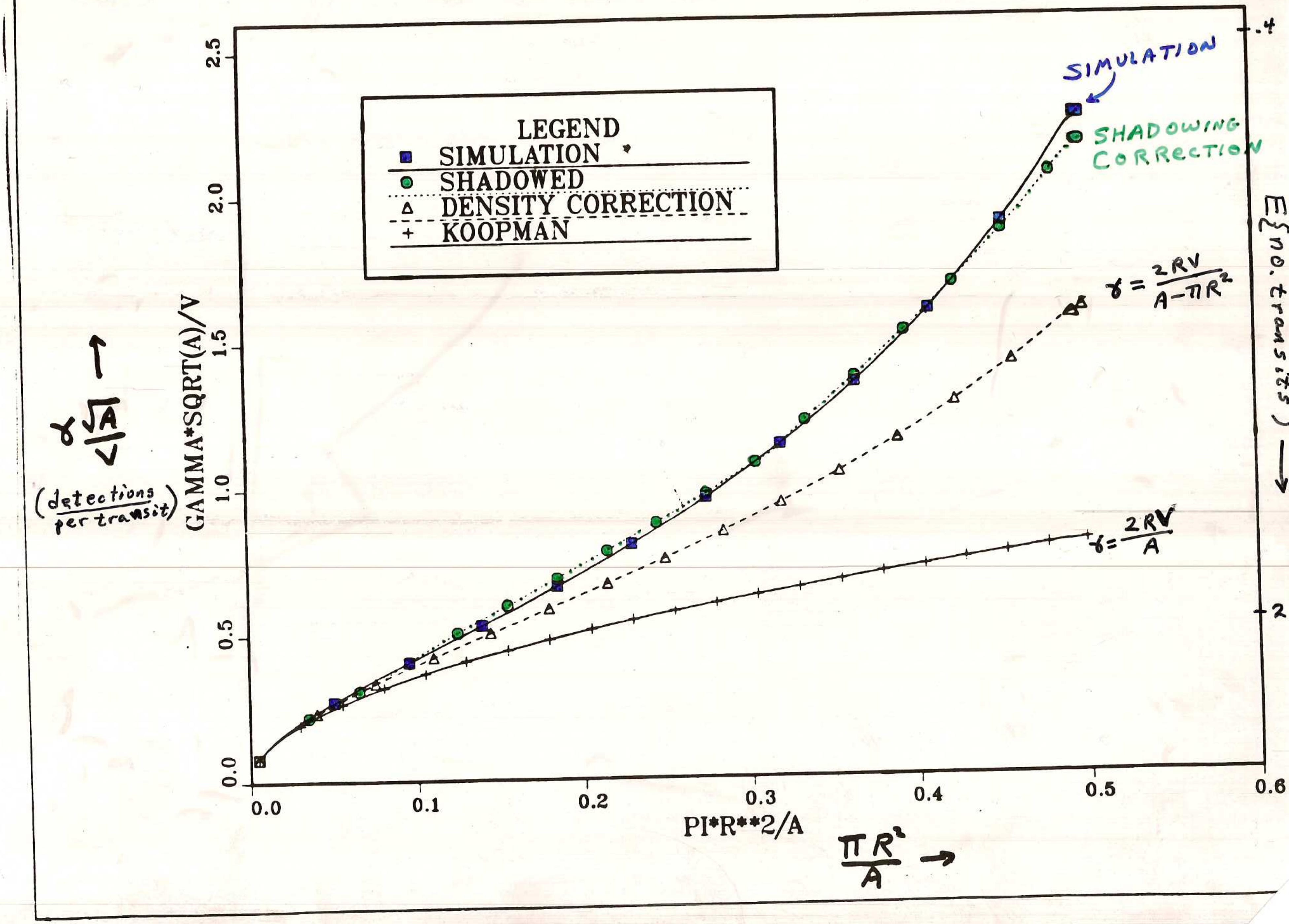
- Simplest correction for (less) shadowing in a lattice, relative to Poisson field
  - -- consider only 4 nearest neighbors (ignore higher order effects)
  - -- use expected values of R_i, R_i^{-1} in 'first try'

- $$\gamma = \frac{2RV}{(A - \pi R^2)}$$ Koopman's, with simplest density correction

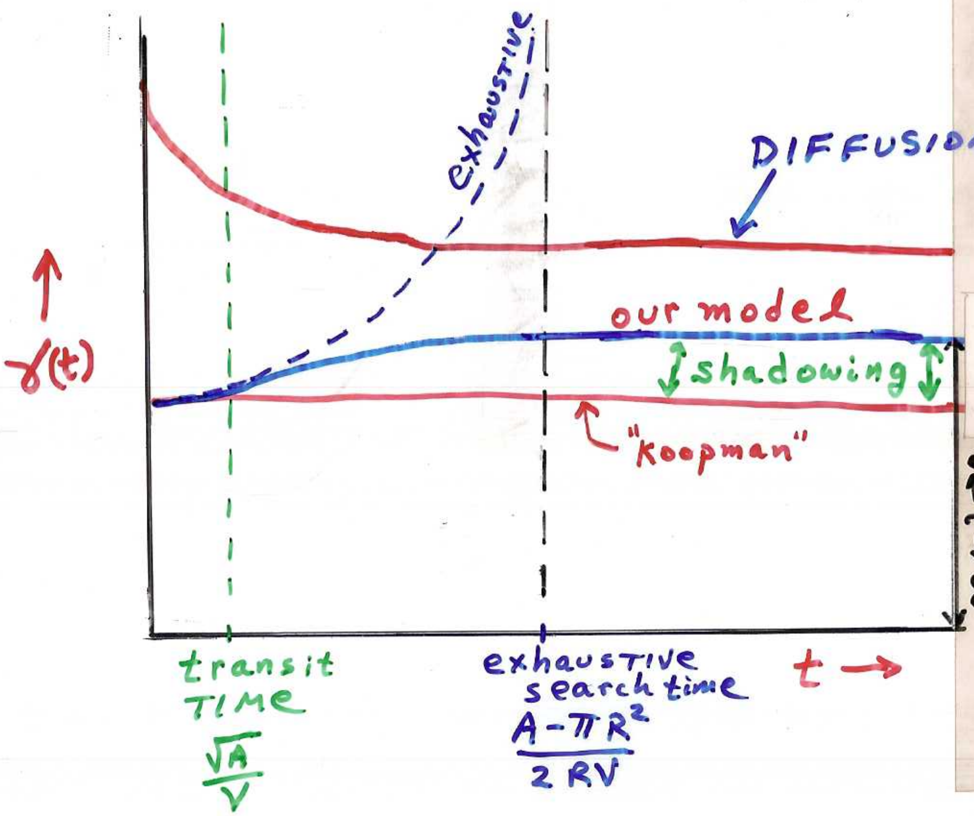
$$\longrightarrow \quad \frac{2RV}{(A - \pi R^2)} \cdot \frac{4 * \text{effective sweep width for lattice}}{\sum_{i=1}^{4} \text{effective sweep width for Poisson field}(\bar{R}_i)}$$

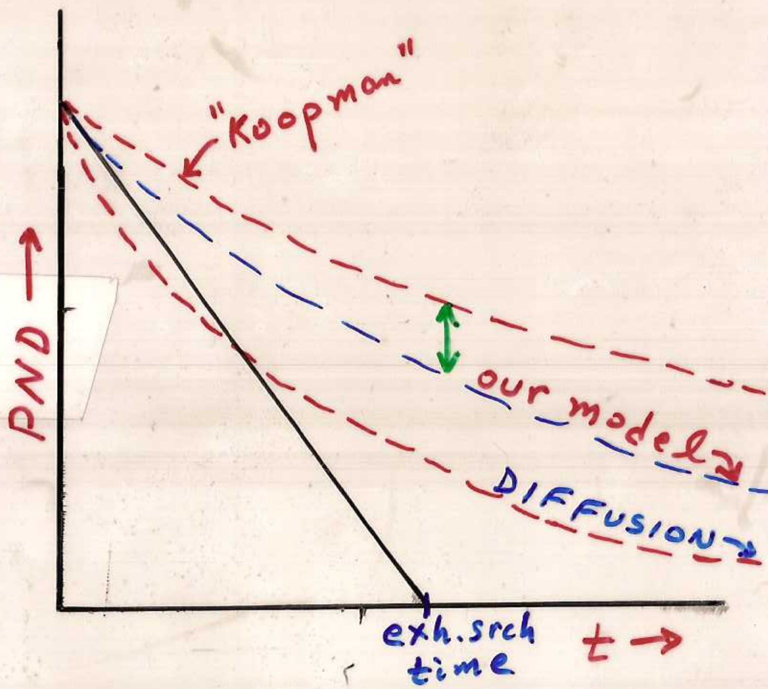
scale up sweep width

NORMALIZED DETECTION RATE



MODEL COMPARISON





- exhaustive search $\gamma(t) = \dfrac{2RV}{A - \pi R^2 - 2RVt}$

- "Koopman" $\gamma(t) = \text{const.} = \dfrac{2RV}{A - \pi R^2}$

- our model "shadowing" $\Longrightarrow$ $\gamma(t) = \gamma_{exh}(t)$, $\left(t \lesssim \dfrac{\sqrt{A}}{V}\right)$; $\gamma(t) \approx \dfrac{2RV}{A - \pi R^2} \cdot \dfrac{1}{\left(1 - \frac{\pi R^2}{A}\right)^{.5}}$

(system doesn't know of boundaries)

CONCLUSIONS

- Neither in Diffusion nor const. $\vec{V}$ Limit is $\gamma(t) = $ cons
- Even for $t >$ "transition time", $\gamma \neq \frac{2RV}{A - \pi R^2}$
  -- $>$ for $\lambda \approx \phi$
  -- $<$ for $\lambda > \lambda^*$
- SPECULAR Reflection $\Longleftrightarrow$ ∞ array of detectors
  -- our 'SHADOWING' CORRECTION $\Rightarrow$ GOOD FIT to SIMULATION
  -- possibly useful for sonobuoy arrays, etc.
- Position of detector in A strongly affects γ
- RANGE OF SITUATIONS $\ni$ RANDOM search approximation is appropriate
  - exponential PND(t)
  - γ (A, R, λ, pos'n, refl. policy, area shape, ...)