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Intensity fluctuations of amplified spontaneous emission in bidirectional laser amplifiers

N. B. ABRAHAM

Department of Physics, Swarthmore College, Swarthmore, Pennsylvania 19081, U.S.A.

and

EDWARD B. ROCKOWER†

Ketron, Inc., Valley Forge Executive Mall, Wayne, Pennsylvania 19087, U.S.A.

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Abstract. Intensity fluctuations of amplified spontaneous emission (ASE) in a bidirectional laser amplifier are analysed using a noise amplifier rate equation model. In contrast to the unidirectional case, in which saturation leads to laser-like intensity statistics, we have found that the saturation, as the mechanism for coupling between the counter-propagating beams, in general leads to a non-zero asymptotic value for the intensity fluctuations. For a symmetric amplifier the normalized variance of the output intensity fluctuations declines with increasing amplifier length from the initial value of unity to a limiting value of approximately 0·277. In contrast, when the noise sources of the two counter-propagating beams have unequal mean intensities, the normalized variance of the stronger beam declines more rapidly with length to less than 0·277 while for the weaker beam it reaches a limiting value greater than 0·277. We also find that there are features of the mode competition which are closely related to those observed in a two-mode ring laser.

1. Introduction

Laser amplifiers have received considerable attention in the context of efforts to understand cosmic masers [1] or the characteristics of optical amplifiers in applications such as optical communications and fusion research. However, previous analyses have faced a number of complicating factors including the finite spectral width of the radiation, the generally extended amplifier dimensions admitting many spatial modes, the existence of counter-propagating beams in even the most confining special case of a narrow cylindrical geometry, the finite amplifier length requiring consideration of propagation effects, and the possibility of atomic coherent phenomena in coupling between the optical signal and the medium.

Early work on laser amplifiers concentrated on the variation of the mean intensity and the spectral profile with amplifier length for both unidirectional and bidirectional cases. These models generally neglected intensity fluctuations and coherent effects and treated the evolution of the intensity via rate equations. The results include such features as gain-narrowing and saturation rebroadening of the spectral

† Present address: Lawrence Livermore Laboratory, University of California, P.O. Box 808, Livermore, California 94550, U.S.A.

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profile (the latter when the initial gain of the medium is inhomogeneously broadened) and the growth of intensity with amplifier length (see for example [2] and [3] and references therein).

Theoretical work on intensity fluctuations has been limited to the consideration of unidirectional systems. Results reported include an early demonstration of 'excess fluctuations' due to coupling between frequency components in the broadband signal [4], analytic treatment of a noise amplifier rate equation without coherent interactions [5, 6] and numerical modelling of a noise amplifier as a sum of discrete spectral components [7].

The other major area of investigation of laser amplifiers (also limited to unidirectional cases) has involved the effects of atomic coherence on pulse propagation and distortion ([5, 6] and references therein). These theories model the spontaneous emission noise as an input of 'thermal pulses' at one end.

No theoretical work has been reported on intensity fluctuations in any model of a bidirectional amplifier aside from early modelling by Klüver (1966) related to experiments on the saturation of mean amplifier noise by an injected signal. With the advent of recent experimental results [8–12] reporting systematic measurements of the fluctuations in such amplifiers, the need for even the most approximate theory has become apparent.

Reported here are preliminary results for a noise amplification model in which the distributed noise is approximated by thermal input noise. We solve the rate equation for the intensities as a type of stochastic differential equation [13] in which the probabilistic aspects are the initial values of the intensities. This approximation is applicable when the growth of the intensity of each beam is primarily a result of gain rather than the distributed noise. These results provide a baseline for comparison with experiments and with more advanced theories as such work is completed.

2. Bidirectional amplifier model

The model used here extends the rate equation methods used successfully by Casperson [2] in a recent treatment of intensity build-up and spectral narrowing and rebroadening in bidirectional laser amplifiers. We consider a narrow cylindrical geometry with a single beam propagating along the axis in each direction. We extend the so-called 'noise amplifier' [5–7] approximation, which replaces the distributed spontaneous emission noise with thermal input signals, to the bidirectional case.

The rate equation assumption (as justified by Casperson [2]) requires that the lifetimes of the atoms (T_1) and the atomic dipole dephasing time (T_2) be short compared with the characteristic time for the intensity variation (T_c). We further assume that the amplifier is short compared with the intensity correlation length of the input noise so that our equations are time-independent, with the fluctuations determined by taking ensemble averages over the noise.

The intensities of the two counter-propagating beams are thus governed by the following equations [2, 5, 6]:

$$\frac{dx^{\pm}}{dz} = \pm\left(\frac{gx^{\pm}}{1+x^{+}+x^{-}} - \alpha x^{\pm}\right), \tag{1}$$

where the intensities, $x^{\pm} = I^{\pm}/I_s$, have been normalized to the saturation intensity.

3. Results

Equation (1) implies the existence of an invariant similar to one used by Casperson [2]: $x^+(z)x^-(z)$ = constant along the amplifier. In particular for an amplifier of length L,

$$x^-(z) = x^+(L)n^-/x^+(z), \tag{2}$$

where we take n^+ and n^- to be the input values for the two beams, $x^+(0)$ and $x^-(L)$ respectively. Physically, this invariant results from the fact that the local gain is the same in each direction.

For comparison with experimentally measurable results we choose to consider here the variation of $x^+(L)$ as a function of the total amplifier length L rather than x^+ as a function of z. Eliminating $x^-(z)$ in equation (1) using equation (2) and integrating from $z=0$ to $z=L$ yields the following implicit functions relating $x^+(L)$ to n^+ and n^-. For the general bidirectional case we find

$$L = -\frac{1}{\alpha}\ln\left(\frac{x^+(L)}{n^+}\right) + \frac{g}{\alpha^2\sqrt{q}}\ln\left[\left(\frac{\kappa - 2x^+(L) - \sqrt{q}}{\kappa - 2x^+(L) + \sqrt{q}}\right)\left(\frac{\kappa - 2n^+ + \sqrt{q}}{\kappa - 2n^+ - \sqrt{q}}\right)\right] \tag{3 a}$$

where $q = \kappa^2 - 4x^+(L)n^-$, and $\kappa = (g-\alpha)/\alpha$. We note the special result for equation (3 *a*) without loss ($\alpha = 0$):

$$gL = \ln\left(\frac{x^+(L)}{n^+}\right) + x^+(L)\left(1 + \frac{n^-}{n^+}\right) - (n^+ + n^-). \tag{3 b}$$

Given $x^+(n^+, n^-, L)$ at L and $x^-(n^+, n^-, L)$ at 0 we can calculate the moments of the distribution of the intensity fluctuations and the cross-correlation between the beams using the formula

$$\overline{[x^-(0)]^n[x^+(L)]^m} = \int_0^\infty \int_0^\infty [x^-(n^+, n^-, L)]^n [x^+(n^+, n^-, L)]^m P^+(n^+)P^-(n^-)\, dn^+ dn^-. \tag{4}$$

We calculate below the mean intensity and normalized variance Q,

$$Q^+ \equiv \frac{\overline{(x^+(L))^2}}{(\overline{x^+(L)})^2} - 1,$$

for various inputs and the cross-correlation function C,

$$C \equiv \frac{\overline{x^+(L)x^-(0)} - \overline{x^+(L)}\,\overline{x^-(0)}}{\sqrt{[\overline{(\Delta x^+(L))^2}\ \overline{(\Delta x^-(0))^2}]}},$$

for equal inputs.

3.1. *Asymptotic results*

The limiting output of the amplifier for increasing L is found when

$$\frac{dx^+(L)}{dL} = 0;$$

in general

$$\frac{dx^+(L)}{dz}$$

is not also zero at the same time.

Applying this condition to equation (3 *a*) yields two solutions

$$x^+(L)=\kappa-n^- \quad \text{and} \quad x^+(L)=(n^+/n^-)(\kappa-n^+). \tag{5}$$

By symmetry, equivalent solutions apply for $x^-(0)$. Using the invariant (equation (2)) and the fact that $0\leqslant x^+\leqslant\kappa$, we see that when $n^+\geqslant n^-$,

$$x^+(L)=(\kappa-n^-) \quad \text{and} \quad x^-(0)=(n^-/n^+)(\kappa-n^-); \tag{6 a}$$

and when $n^+\leqslant n^-$,

$$x^+(L)=(n^+/n^-)(\kappa-n^+) \quad \text{and} \quad x^-(0)=(\kappa-n^+). \tag{6 b}$$

Using these analytic relationships for $x^+(L)$ in equation (4) and assuming negative exponential distributions for n^+ and n^- characteristic of gaussian noise, we find the following asymptotic values:

$$\overline{x^+(L)}=\kappa\gamma^{-1}\ln(1+\gamma), \tag{7}$$

$$Q^+=\frac{2\gamma-2\ln(1+\gamma)}{[\ln(1+\gamma)]^2}-1, \quad \text{and} \tag{8 a}$$

$$C=\frac{1}{\sqrt{(Q^+Q^-)}}\left(\frac{\gamma^{-1}(1+\gamma^2)\ln(\gamma+1)-\gamma\ln\gamma-1}{\ln(1+\gamma)\ln(1+\gamma^{-1})}\right)-1, \tag{8 b}$$

where we assume $\kappa\gg n^\pm$, and $\gamma=\overline{(n^-)}/\overline{n^+}$. When $\gamma=1$, specific values which are appropriate to ASE are $\overline{x^+}(L)=\kappa\ln 2$, $Q^+=0{\cdot}277$ and $C=-0{\cdot}707$. When $\gamma\gtrsim 5$, Q^+ becomes greater than one. Since the intensity is much larger than the noise in this case, this prediction of output intensity fluctuations greater than those characteristic of thermal noise might be confirmed experimentally.

For the special case when $\alpha=0$, we see that for large $x^+(L)$ equation (3 *b*) asymptotically approaches the relation

$$x^+(L)=gL(1+(n^-/n^+))^{-1}. \tag{9}$$

From this equation we see that when there is no damping the mode competition in a bidirectional laser amplifier gives rise to the same effects as those found elsewhere for the two-mode ring laser (see equation (5) in [14]). In particular when n^+ and n^- have negative exponential distributions with the same mean, $x^+(L)$ has a uniform distribution with $Q^+=\frac{1}{3}$, and $C=-1$ for all γ, in contrast to the values found following equation (8 *b*).

3.2. *Numerical results*

Interpretation of laboratory experiments requires that we investigate the intensity fluctuations for finite amplifier lengths. The mean intensity and normalized variance of the output beam as functions of the total amplifier length can be calculated using equations (3 *a*) and (4). When taking $P^+(n^+)$ and $P^-(n^-)$ to be negative exponential distributions we can perform the integration by Gauss–Laguerre† quadratures. A modified binary search was used to find the value of x^+ satisfying equation (3 *a*) for each pair of values for n^+ and n^-.

The results are displayed in figures 1–4. The numerical results for large L agree well with values calculated from the analytic relations (equations (7) and (8)). The figures show the striking difference between the unidirectional amplifier ($\overline{n^-} \ll \overline{n^+}$) and the bidirectional amplifier. In general, the competition between the two beams sustains a level of intensity fluctuations much higher than that characteristic of the single beam case. When $\overline{n^-} \ll \overline{n^+}$ the intensity evolves to laser-like statistics with $Q^+ = 0$ as expected. In the opposite extreme, $\overline{n^+} \ll \overline{n^-}$, our model predicts values of $Q^+ > 1$. From figure 1 we also see that the use of equal thermal noise imputs changes the mean intensity only slightly from the result for coherent imputs.

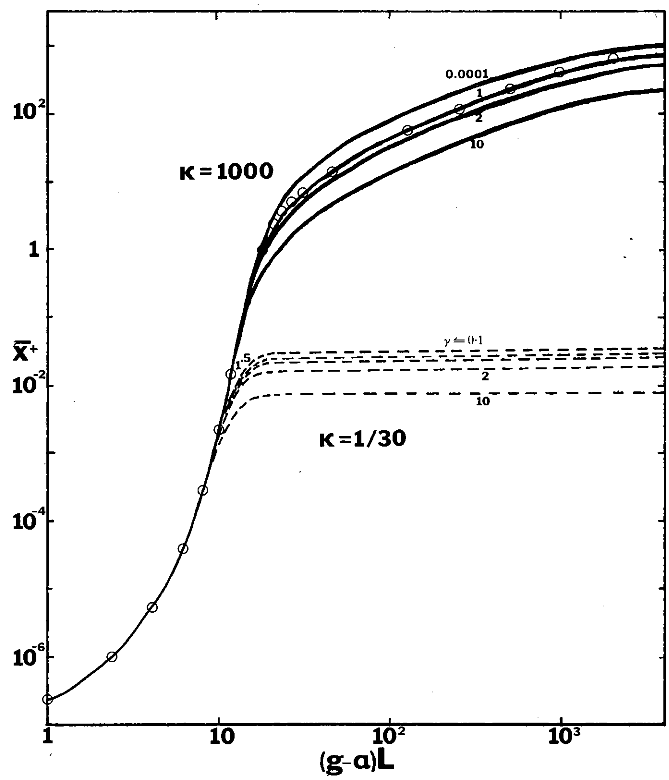
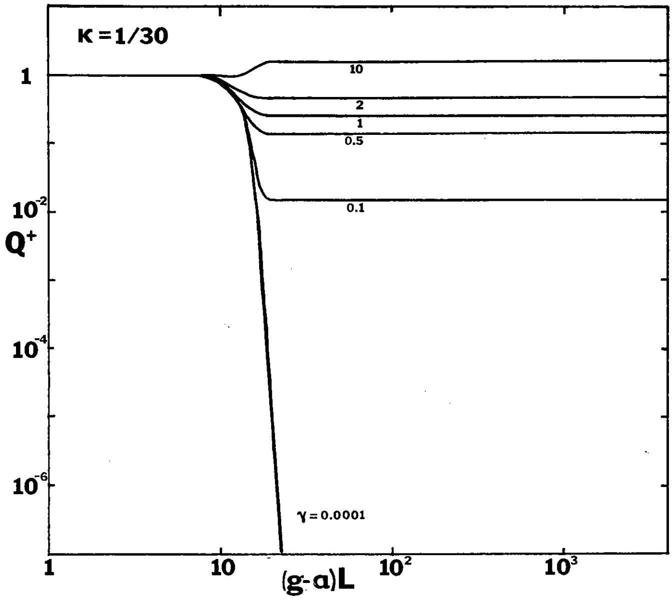


Figure 1. Mean intensity, $\overline{x^+}$, versus small signal gain lengths for $\kappa = 1/30$ (–––) and $\kappa = 1000$(———). In each case we assume negative exponential distributions for n^+ and n^- and take $\overline{n^+} = 10^{-7}$; $\gamma(\overline{n^-}/\overline{n^+})$ was varied as shown. Values for coherent (rather than thermal) input when $\gamma = 1$ and $\kappa = 1000$ are indicated by (○).

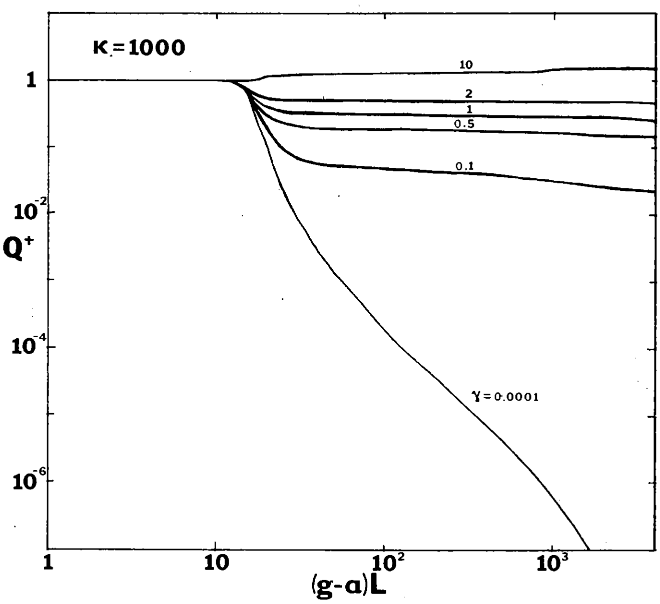
† We use the cartesian direct product formula [15]:

$$\iint F(y, z) \exp(-y) \exp(-z)\, dy dz = \sum_{ij}^{N} F(x_i, x_j) w_i w_j,$$

where the roots (x_i) and weights (w_i) are tabulated by Stroud and Secrest [16]. For our final calculations, that choice of N was made which seemed to give 1 per cent accuracy as determined by extrapolation from results for smaller values of N. This approximate accuracy was confirmed by comparison with the analytic formula for the asymptotic values.

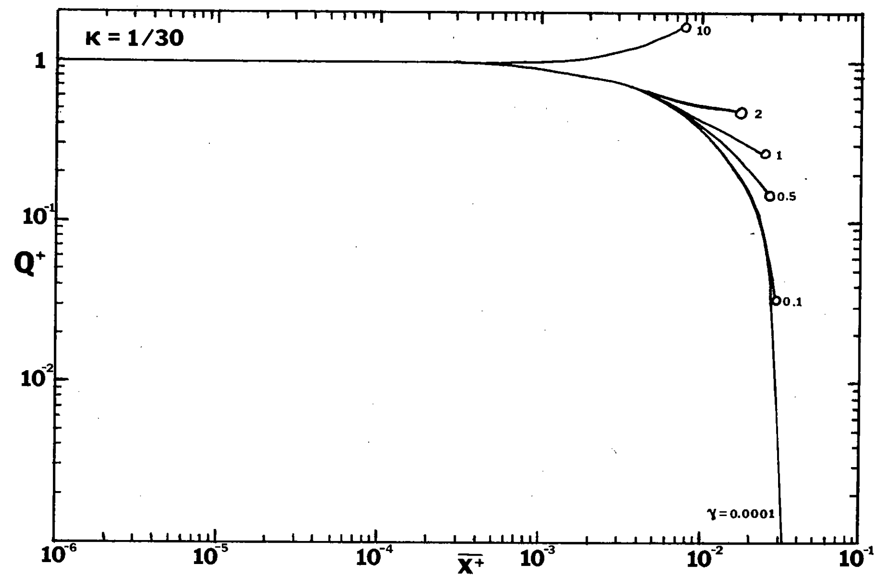


(a)

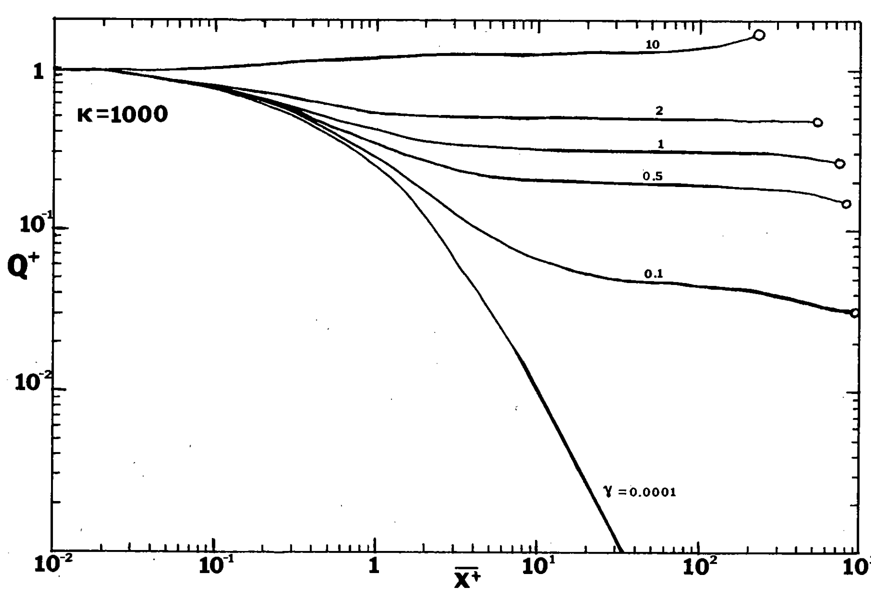


(b)

Figure 2. Normalized variance versus small signal gain lengths for $\kappa=1/30$ (*a*) and $\kappa=1000$ (*b*).



(*a*)



(*b*)

Figure 3. Normalized variance versus mean intensity for $\kappa = 1/30$ (*a*) and $d\kappa = 1000$ (*b*). The end point indicated on each curve, marked by (○), indicates the asymptotic values.

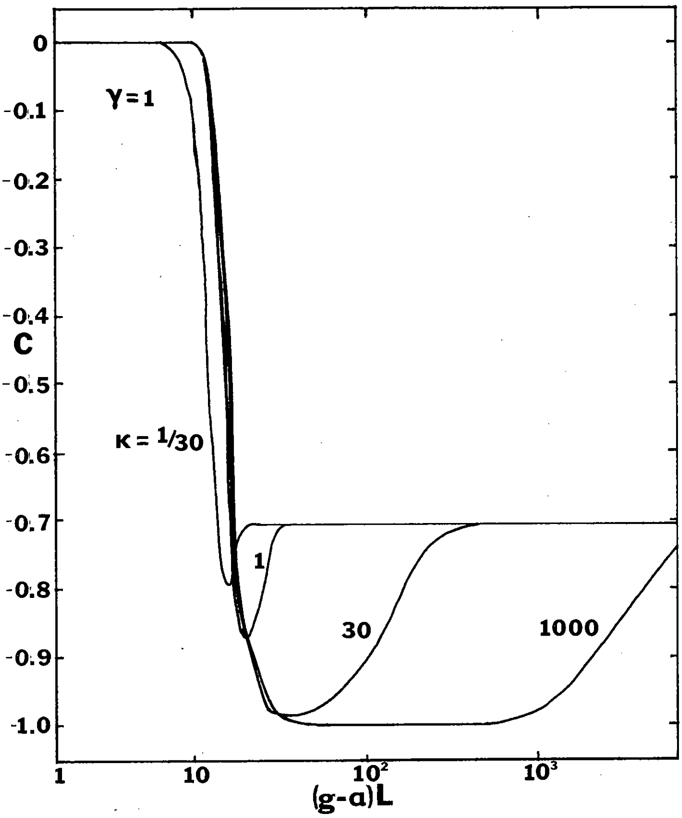


Figure 4. Cross-correlation function versus small signal gain lengths for a symmetric amplifier ($\overline{n^+} = \overline{n^-} = 10^{-7}$).

Notice the points of inflection of the curves in figure 3 (*b*), where $\kappa = 1000$. Here both Q^+ and C (see figure 4) have evolved to the values characteristic of the steady-state two-mode ring laser [14]. With damping the gain ultimately goes to zero and the normalized variance declines to the asymptotic values calculated from equation (8 *a*). The minimum value of the cross correlation, -1, indicates maximal anticorrelation for intermediate amplifier lengths. C then increases implying that the fluctuations in the beams become more independent as the amplifier is lengthened.

4. Discussion

No experimental measurements of the ASE intensity fluctuations are presently available in systems for which $T_c \gg T_1 \geqslant T_2$, which is necessary for the validity of equation (1). However, when $T_1 > T_c > T_2$ a set of rate equations apply, although not in as simple a form as equation (1), since the population inversion does not adiabatically follow the intensity fluctuations. For this latter case measurements are available [10–12] which show qualitative agreement with our predictions of a slow variation of Q with intensity and the generally finite asymptotic value of Q. Those results for the case when $T_1 \geqslant T_2 \geqslant T_c$ [8–12] appear qualitatively different from the predictions of our theory, as expected since no rate equation approximation is valid there.

We have compared models of the bidirectional amplifier and the two-mode ring laser. The results have been shown to be remarkably similar, particularly when Q reaches the ring laser steady-state value, at which point the maximum anticorrelation of the beams is attained. This similarity occurs in spite of the fact that in the ring laser the two modes both increase in time from initially weak spontaneous noise, while in the amplifier each beam increases in intensity in the direction of its travel and thus has its maximum intensity where the other is at a minimum.

This model can also be used to calculate the effect of counter-propagating ASE on other signals, such as the coherent output of a laser, injected in one end. Generalized inputs and other extensions of the model are currently being investigated.

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Mit Hilfe eines Ratengleichungsmodells für Rauschverstärker werden Intensitätsfluktuationen verstärkter spontaner Emission in einem in beide Richtungen betriebenen Laserverstärker analysiert. Im Gegensatz zum Betrieb in nur einer Richtung, bei dem es durch Sättigungserscheinungen zu laserähnlichen Intensitätsfluktuationen kommt, haben wir festgestellt, daß die Sättigung als Kopplungsmechanismus gegenläufiger Lasermoden im allgemeinen zu einem nicht verschwindenden asymptotischen Wert für die Intensitätsfluktuation führt. Beim symmetrischen Verstärker nimmt die normierte Varianz der Intensitätsfluktuation am Ausgang mit zunehmender Verstärkerlänge vom Anfangswert 1 ab auf einen Grenzwert von etwa 0,277. Im Gegensatz dazu nimmt bei ungleicher Stärke der Rauschquellen der gegenläufigen Moden die normierte Varianz des stärkeren Modus mit zunehmender Länge stärker ab, und zwar auf weniger als 0,277, während der schwächere Modus einen Grenzwert größer als 0,277 erreicht. Wir haben ferner Eigenschaften dieser konkurrierenden Moden-Wechselwirkung gefunden, die zu jenen im Zweimoden-Ringlaser engen Bezug haben.

References

[1] Cook, A. H., 1977, *Celestial Masers* (London: Cambridge University Press).

[2] Casperson, L. W., 1977, *J. appl. Phys.*, **48**, 256.

[3] Parks, J. H., 1973, *Esfahan Symposium on Fundamental and Applied Laser Physics*, edited by M. Feld, A. Javan and W. Kurnit (New York: Wiley–Interscience), pp. 867–894.

[4] Gamo, H., 1966, *I.E.E.E. Jl quant. Electron.*, **2**, xii.

[5] Hopf, F., 1974, *PHQE: High Energy Lasers and Their Applications*, edited by S. Jacobs, M. Sargent and M. O. Scully (New York: Addison-Wesley), pp. 77–176.

[6] Hopf, F., 1975, Optical Sciences Center Report.

[7] Menegozzi, L., and Lamb, W. E., Jr., 1978, *Phys. Rev.* A, **17**, 701.

[8] Gamo, H., and Chuang, S. S., 1971, AFOSR Final Report, Contract No. F19628-70-C-0101.

[9] Gamo, H., and Chuang, S. S., 1973, *Coherence and Quantum Optics*, edited by L. Mandel and E. Wolf (New York: Plenum), p. 491.

[10] Gamo, H., Osada, H., and Liao, A. S., 1976, *J. opt. Soc. Am.*, **66**, 1076.

[11] Gamo, H., Osada, H., and Liao, A., 1978, *Coherence and Quantum Optics*, Vol. IV, edited by L. Mandel and E. Wolf (New York: Plenum), pp. 611–622.

[12] ABRAHAM, N. B., and SMITH, S. R., 1978, *Coherence and Quantum Optics*, Vol. IV, edited by L. Mandel and E. Wolf (New York: Plenum), pp. 623–633.
[13] VAN KAMPEN, N. B., 1976, *Physics Lett.* C, **24,** 171.
[14] ROCKOWER, E. B., and ABRAHAM, N. B., 1979, *Optica Acta*, **26,** 1289.
[15] STROUD, A. H., 1971, *Approximate Calculations of Multiple Integrals* (Englewood Cliffs: Prentice-Hall).
[16] STROUD, A. H., and SECREST, D., 1966, *Gaussian Quadrature Formulas* (Englewood Cliffs: Prentice-Hall).