GENERALIZED CHERENKOV RADIATION
FROM TACHYONIC SOURCES

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CHAPTER 1

Introduction

During the period preceding the publication by Einstein of his paper on the theory of relativity, a number of authors considered electromagnetic phenomena associated with sources travelling at velocities greater than that of light.[1] These considerations were neglected after the impossibility of accelerating a particle beyond the speed of light was shown by Einstein in 1905. Ironically, the first observation of the electromagnetic emission characteristic of charges moving faster than the phase velocity of light in a medium was apparently made by Madame Curie only five years later, [58] Jelley. The nature of this blue-white Cherenkov radiation was not recognized until extensively studied by Vavilov, Cherenkov and Frank & Tamm in the thirties. [34] Cherenkov, [37] Frank + Tamm, [58] Jelley, [60] Cherenkov, [60] Tamm.

We note that Cherenkov radiation can be viewed from a more general point of view. For example, we consider the various Cherenkov effects that might occur if the propagation velocity of a field (or the limiting velocity of a particle) were increased or decreased because of general field-field or field-particle interactions.

In the event that the propagation speed of a field ϕ exceeds the Einstein speed c any other field Ψ with which ϕ couples might be Cherenkov radiated by ϕ. If a ϕ's propagation velocity is decreased with respect to c (as happens with light in ordinary Cherenkov radiation) then there is the possibility that ϕ itself will be Cherenkov radiated by other particles or fields ψ with which it couples.

Cherenkov effects are also likely to occur in situations in which particle speeds <u>always</u> exceed the Einstein speed c. Renewed interest has been awakened in this last possibility, in the past decade or so, particularly by the articles of Bilaniuk, Deshpande and Sudarshan (1962) (hereafter BDS) and Feinberg (1967).

In this thesis we will be particularly concerned with this last case in which the particle speed is always greater than the speed of light in a vacuum.

In chapter II certain anticipated properties of faster than light particles (dubbed "tachyons by Feinberg) will be reviewed. Also we will take notice of the difficulties presented by the localizability, instability and unitarity problems of tachyon theory, and of the paradoxes associated with the possibility of signals traveling backward in time. A number of experiments that have been performed to try to detect tachyons in spite of the unresolved problems are then described. These include missing mass, Cherenkov radiation and extensive air shower searches.

In chapters III and IV we will extend our considerations to those aspects of tachyon behaviour which may be particularly relevant to the detection of such particles. The property of tachyon interactions which is singularly characteristic is that of Cherenkov radiation of other particles from tachyonic sources Since this emission may be quite important to many aspects of tachyon behaviour (e.g., range, trajectory, detectability, and the resolution of paradoxes), we analyze in detail the generalized Cherenkov emission of a massive (normal) field.

Initially certain aspects of the motion of tachyons will be derived on the basis of a classical single particle picture of a superluminal object acted on by various forces. Some Lorentz transformation properties of the dynamical variables of the tachyon, such as that of its acceleration, are discussed. The counterintuitive relation of the direction of the acceleration with reppect to the direction of the force, found qualitatively by BDS ~~[illegible]~~, is derived quantitatively. We then show the relation of these results to Cherenkov radiation. Also we present trajectories of a charged tachyon in certain electric and magnetic fields relevant to experiments already performed.

From this classical model there are then two transitions which must be effected. The first transition is to introduce a free quantized field which has the classical particle tachyonic motion as a limit of wave packets that arise on taking appropriate field operator matrix elements. We may then look at classical fields (the matrix elements of the free quantum fields) whose Fourier components obey the energy-momentum dispersion relation characteristic of tachyons, i.e.

$$E^2 = (\bar{p})^2 c^2 - (mc^2)^2$$

It is natural to require that both E and $\bar{p}$ be real (theory of type I), but that implies that $|\bar{p}| \geqslant mc$, which leads to severe difficulties in the localization of tachyons [67] Feinberg, [69] Peres. In theories of type II one allows all real values of $\bar{f}$, thus avoiding localization problems, but the associated imaginary E's produce severe instability problems. Furthermore theories of type II do not possess the Lorentz transformation properties we would like any physical theory to have. Unfortunately it turns out that free theories of type I also fail to have the desired Lorentz transformation properties, so we are forced to make the second transition in order to reacquire those properties.

The second transition is to introduce two kinds of interactions: one kind which creates and destroys tachyons in such a way that no tachyon ever escapes to spatial infinity (thus solving the Lorentz covariance problem), and a second kind in which tachyons are (massive or massless) Cherenkov sources for ordinary matter. For electrically charged tachyons there is a radiation reaction force produced by its Cherenkov radiation. We will show that there is the possibility that more general types of interaction could give rise to a Cherenkov radiation of massive fields. Hence the two transitions: 1) from the classical particle tachyons to quantum fields with localization and Lorentz covariance problems; 2) from externally applied forces to radiation reaction forces from a generalized Cherenkov radiation.

The modification of the trajectory produced by the evolution of the tachyon's radiative environment under these conditions will then be found. A new type of hyperbolic motion in configuration space will be shown to correspond to a certain invariant rate of evolution of each k component of the tachyon wave packet over its mass hyperboloid in k space. This motion on the mass hyperboloid will be found through appeal to Lorentz covariance considerations. The configuration space evolution of the radiating tachyon (relevant to the classical particle picture), can be deduced from the momentum space evolution.

The characteristics of the radiated field are investigated next. The energy which is radiated into the field is found to agree with the tachyon energy loss formula found in the preceding sections. In order to calculate that energy loss and the features of the radiation we pursue two models. Not surprisingly, certain simplifications arise if we first treat the tachyon as a prescribed classical source. In the case that the source velocity is held constant certain characteristics of the massive Cherenkov radiation may be derived by looking at the resonant coupling of scurce to field. That is, we first look at the Fourier transform of the charge. In this way we find the relation of angle to energy of emitted massive particles. In contrast to ordinary Cherenkov radiation there is a range of angles of emission, including the forward direction. We also find a minimum value of the wave vector depending on the mass of the radiated field. We next perform a more detailed calculation of the radiated massive field (which we call the "pi" field). We derive the energy radiated per unit time as a function of angle and frequency for a general charge distribution. In this model we also derive the pi radiation emitted at the creation and destruction events terminating the world-line of the tachyon, in analogy with radiation emitted during beta decay (inner bremsstrahlung)[62] Jackson.

That there can be Cherenkov radiation of massive fields, is significant in any attempt to detect tachyons produced through strong interactions. However, we find that for certain charge distributions this potential radiation might be inhibited or even completely suppressed.

We wish to take account of the effect of the hyperbolic motion of the tachyon on the Cherenkov radiation. Therefore, in an appendix we determine the wave front of the Cherenkov radiation using Leibnitz's method for finding the envelope of a family of surfaces.

Using the results above as a guide, we then derive the transition rate for emission of generalized Cherenkov radiation in a quantum field theoretic model allowing for recoil of the tachyon. The assumption is made that there is no interference of consecutive emissions. The model is that of quantized tachyon and pi fields interacting through a particular scalar interaction. The interesting difficulties associated with a superluminal form factor are treated in an appendix. Beyond a certain point the two models of generalized Cherenkov radiation converge; once the basic energy loss rate is derived the analysis proceeds the same whether derived using the classical or quantum fields as the source.

Some considerations relevant either to general Cherenkov radiation or to other tachyon phenomena are discussed in the appendices.

The analysis of those parts of experiments which our results affect is made in chapter V. It is shown that some of the experimental conclusions do not reach as far as their authors have stated. In particular, the limits presented in the literature on the cross-sections for production of various types of tachyons are not justified.

Certain suggestions based on our findings are made for further experiments designed to search for tachyons.

Chapter II

In this Chapter we provide a review of the theory and of the experimental searches for faster than light particles. The theory is divided between classical considerations, largely based on an extension of the special theory of relativity, and quantum mechanical aspects. The predictions of Cherenkov radiation of massless electromagnetic and gravitational fields are briefly mentioned. Paradoxes which arise in any consideration of faster than light signals are discussed. Attempts to detect tachyons are then examined with a view to later discussion in the light of our results.

In 1905 Einstein wrote "...velocities greater than that of light have no possibility of existence," [23] Lorentz. To see why he, and afterward almost all others, believed this it should be remembered that in the view of classical mechanics a particle attains a velocity v after being accelerated continuously through all intervening velocities. According to the relation between energy and velocity which Einstein obtained:

$$E = \frac{mc^2}{\sqrt{1-v^2/c^2}} \tag{1}$$

it would require an infinite amount of energy to accelerate a particle to c. However, with the advent of quantum mechanics we have become accustomed to the idea of creating a particle already traveling at a given velocity, just as photons always are created with velocity equal to c. Hence it was suggested by Bilaniuk, Deshpande and Sudarshan in 1962 in a classical framework and Feinberg in 1967 in a quantum framework that a third class of particle might exist, with v always greater than c. Just as ordinary particles are excluded from ever reaching velocities equal to c from below, so the tachyons have an infinitely high energy barrier limiting them to v always greater than c.

Relation (1) has been generalized to tachyons by speaking of an imaginary "proper mass" $m_\tau = i|m_\tau|$ so that equation (1) becomes:

$$E = \frac{m_\tau c^2}{\sqrt{\frac{v^2}{c^2} - 1}} \tag{2}$$

It might be preferable to derive this without speaking of imaginary or "proper" quantities. We do this by starting with the usual relation: (set c = 1)

$$E^2 = \bar{p}^2 + m^2 \tag{3}$$

We see from this that we always have $|\bar{p}| < E$ and, since

$$\bar{v} = \frac{\partial E}{\partial \bar{p}} = \frac{\bar{p}}{E} \quad , \tag{4}$$

v is always less than c for ordinary particles. If (3) is generalized to the case of spacelike four momentum; i.e.

$$E^2 = |\bar{p}|^2 - m_\tau^2 \tag{5}$$

where m_τ is still a real, positive quantity then we have $E < |\bar{p}|$ and hence:

$$|\bar{v}| = \frac{\bar{p}}{E} > c$$

Now if we divide equation (5) by E^2 and use (4), we obtain:

$$1 = v^2 - \frac{m^2}{E^2}$$

Hence, solving for E and choosing positive energy,

$$E = \frac{m_T}{\sqrt{v^2/c^2-1}}$$

The possible values of E, $\bar{p}$ lie on the mass hyperboloids, either (3) or (5). The difference due to the change in signs can be seen in figure 1. For ordinary particles, the restriction to the upper part of the hyperboloid, i.e. positive energy, is Lorentz invariant. However, for the hyperboloid of the tachyon it is possible to change the sign of the energy by a suitable Lorentz transformation. The Lorentz transformation of E is:

$$E' = (E - \bar{p}\cdot\bar{u})\gamma \tag{6}$$

where

$$\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$$

and using (4)

$$E' = E\left(1 - \frac{\bar{v}_T\cdot\bar{u}}{c^2}\right)\gamma \tag{7}$$

Now, since $v_T > c$ we can have:

$$\frac{\bar{v}_T\cdot\bar{u}}{c^2} > 1 \tag{8}$$

for $|\bar{u}|<c$, which is an allowable Lorentz transformation. This causes the energy to become negative. In fact, if (8) were an equality we would have E' = 0, and hence $v_T = \infty$. The possibility of negative energies seems to open the vista of infinite sources of energy and instability against continual emission of tachyons by ordinary matter. However there is another difficulty associated with velocities greater than that of light. The Lorentz transformation of a time interval associated with a

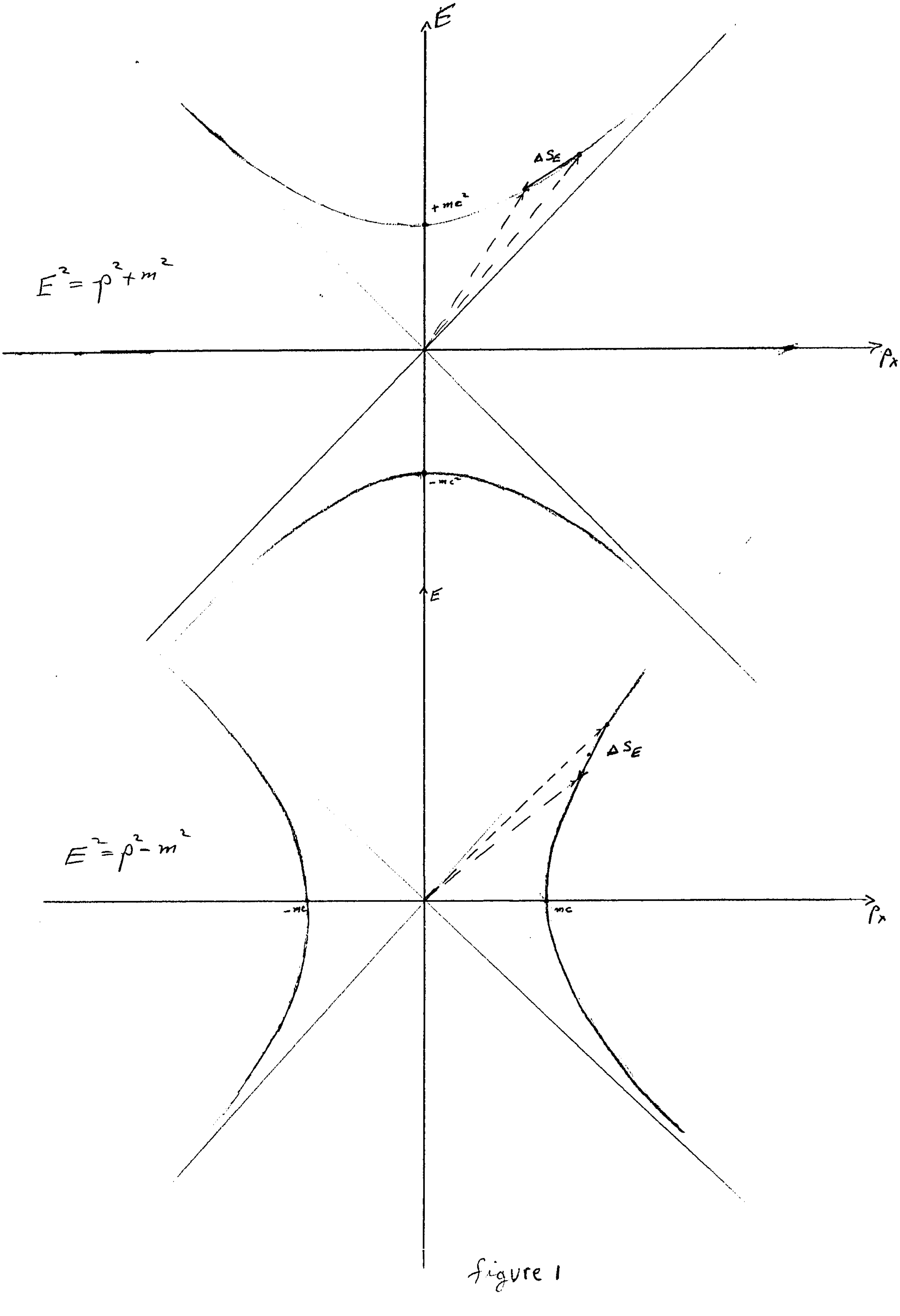


figure 1

tachyon trajectory is:

$$\Delta t' = \left(\Delta t - \frac{\Delta \bar{x} \cdot \bar{u}}{c^2}\right)\gamma \qquad (9)$$
$$= \Delta t \left(1 - \frac{\bar{v}_r \cdot \bar{u}}{c^2}\right)\gamma$$

Note that this quantity can also change signs under a Lorentz transformation. This is just the observation, basic to relativity theory, that earlier and later are not invariant relations for events separated by a spacelike interval. BDS noted that the condition for the sign change of E is the same as the sign change of Δt . They have proposed a resolution of the difficulties in terms of a "reinterpretation principle". In considering any process involving tachyon worldlines one interprets a negative energy tachyon going backward in time, as a positive energy tachyon going forward in time in the opposite direction. They characterize this as being "anti-parallel" to the Stückelberg-Feynman interpretation of positrons as negative energy electrons going backward in time. Different observers will then have dissimilar descriptions of any process which involves tachyons. It is conceivable, however, that experimentally testable relations will transform correctly under changes of Lorentz frame, even though the descriptions of unobservable intermediate processes do not exhibit manifest Lorentz covariance.

One example of these dissimilar descriptions is given by Feinberg. He describes two atoms: one initially excited, the other in its ground state. In the inertial frame in which the atoms are originally at rest the excited atom decays with emission of a positive energy tachyon. Subsequently the tachyon is absorbed by the second atomewhich then jumps into its excited state. Each of the atoms recoils of course, because of the emission or absorption of the tachyon. Our second observor's laboratory frame is such that the atoms appear to be moving with velocity directed parallel to the vector from the initially unexcited, to the excited atom. His velocity is such that equation (8) is satisfied. Hence, the tachyon energy is negative and the time at which the negative energy tachyon is absorbed appears to precede the time at which it is emitted. This observor reinterprets this as the following: the unexcited moving atom in its ground state transforms some of its kinetic energy into a positive energy tachyon and the energy needed to jump into an excited state. Later the positive energy tachyon is absorbed by the other excited atom which loses its energy of excitation, but losing kinetic energy, recoils sufficiently to conserve energy. We will later show that this novel situation has an analog in a medium. Also, if the tachyon is charged it must be "reinterpreted" as an antitachyon.

Considering the geometric Huygens construction for the angle of emission of Cherenkov radiation ([58] Jelley), and the fact that a particle with spacelike four momentum is kinematically allowed to decay into itself plus a photon, Bilaniuk, Deshpande and Sudarshan suggested that tachyons might be found by searching for this radiation. This has motivated both experiments and theoretical investigations into the detailed properties of such radiation. In addition, the possible existence of Cherenkov radiation of the gravitational field was investigated for the same reasons. Although the possibility of Cherenkov radiation of massive fields has also been suggested previously, it has not been followed up except in this thesis so far as we know.

It has been observed from equation (2) that a tachyon will speed up as its energy decreases, which will happen if the tachyon radiates. Hence a force and the associated acceleration can be in opposite directions. Because infinite velocity and zero energy are not Lorentz covariant notions, we conclude that the tachyon will eventually acquire negative energy. When it is in this state it will be reinterpreted as an incoming positive energy antitachyon which annihilates the original tachyon at the zero-energy point. The fact that this limits one's freedom in specifying the initial conditions for even classical tachyons was pointed out by [72] Jones and others.

It will probably be an essential feature of tachyon physics that it is impossible to control by external agencies the processes of emission and absorption of tachyons: we cannot suppose that we may choose to emit a tachyon or not as we please. The The Cherenkov emission of gravitational radiation has been calculated, but not in a Lorentz covariant manner, ([72] Lapedes and Jacobs). Jones [72] did obtain a covariant form for the energy loss due to electromagnetic Cherenkov radiation.

There have also been arguments to the effect that there will be no Cherenkov radiation from electrically charged tachyons. These considerations are based on what is called a "Generalized Lorentz Transformation". Some authors attempt to establish an almost perfect symmetry between ordinary and superluminal "inertial frames" ([74] Mignani and Recami). In this hypothetical superluminal world, the laws of physics are the same as in our own world. As a consequence, in addition to forbidding Cherenkov radiation, they suggest the possibility that an "electric charge " in the tachyon inertial frame transforms to a magnetic monopole under such generalized Lorentz transformations. Quantum theories for particles having spacelike four momentum have been investigated. [60] Tanaka, [67] Feinberg, [68] Arons and Sudarshan, [68] Dhar and Sudarshan, [70] Ecker. The negative mass squared (we take m real) Klein-Gordon equation

$$\left(\partial_t^2 - \nabla^2 - m^2\right)\phi = 0$$

has the basic solutions

$$\frac{1}{(2\pi)^{3/2}} e^{\pm ikx}$$

where we use the time-favored Minkowski metric. Hence, $kx = \omega t - \bar{k}\cdot\bar{x}$ and since we use units in which $\hbar = c = 1$ ω satisfies

$$\omega = \left(\bar{k}^2 - m^2\right)^{1/2}$$

In order for the energy to be real, we require that

$$|\bar{k}|^2 \geq m^2$$

Because of this restriction on $\bar{k}$, thus selecting a type I theory, the solutions do not form a complete set on the t = 0 hypersurface, i.e. we cannot form a delta function $\delta^{(3)}(\bar{x}-\bar{y})$ by superposing our basic solutions with t set to zero. We therefore lack some spatial Fourier components necessary to satisfy arbitrary initial conditions for the tachyon field. In the quantized theory we cannot obtain canonical commutation relations for the same reason.

From the incompleteness of the range of k it has been concluded that the tachyon wave packet is nonlocalizable, [69] Peres, [67] Feinberg. Peres says that the obvious generalization of the Newton-Wigner position operator is:

$$\bar{x} = i\left[\frac{\partial}{\partial \bar{k}} - \frac{1}{2}\frac{\bar{k}}{[\bar{k}^2 - m^2]}\right]$$

If the momentum space wave function $f(\bar{k})$ is nonzero at $\bar{k}^2 = m^2$, then xf does not belong to the Hilbert space of square integrable functions. Since $f(\bar{k})$ will be nonzero at $\bar{k}^2 = m^2$ for all invariant functions f but a set of measure zero (to be shown in a moment), the operator $\bar{x}$ is not densely defined, and is therefore useless. In configuration space, he shows that because of the absence of small $|\bar{k}|<m$ the wave function:

$$\psi(\bar{x},t) = \int \exp\left[i(\bar{k}\cdot\bar{x} - Et)\right] f(\bar{k}) \frac{d^3\bar{k}}{[\bar{k}^2 - m^2]^{1/2}}$$

decreases very slowly (at best as $r^{-3/2}$). It makes no sense to single out the class of wave functions with f(k) vanishing at $\bar{k}^2 = m^2$ since this class is not defined by a Lorentz invariant condition unless f(k) is identically zero. This should be compared with the assertion of Feinberg that although tachyon wave packets cannot be made to vanish outside a finite region, they can be made to fall off with an arbitrary power of r. We show that the latter statement, although not covariant, may be true in certain inertial frames.[2]

The little group of the inhomogeneous Lorentz group for space-like four momenta is the group SO(2,1) of rotations in three dimensional pseudo-Euclidean space which leaves invariant the tachyon four momentum in the standard frame such that:

$$p^\mu = (0; 0, 0, m)$$

This group consists of rotations about the tachyon velocity (here taken to be the z direction) and Lorentz boost transformations in the x-y plane. Since this group is non-compact, all representations are infinite dimensional in the spin variable except for the (unitary) spin 0 representation, [58] Shirokov.

Considering the spin 0 case, Feinberg [67] concludes that the tachyon field must be quantized as Fermions, i.e. with anti-commutators. In his theory the vacuum is not invariant since a Lorentz transformation adds an infinite real number to the momentum operator. Feinberg's quantization scheme was intended to get rid of the negative energies arising from Lorentz transformations. However, Arons and Sudarshan [68] suggest incorporating these negative energy states into a Fock space, but insist that the only physically meaningful quantities are the transition amplitudes to which one applies the "Reinterpretation Principle" of BDS. Thus negative energies are eliminated by "reinterpretation" at the end.

Dhar and Sudarshan [68], considering spin 0 Bosons and dealing with the negative energy states in the same way, also consider interactions. They conclude that the scattering amplitudes may be calculated by substituting ($-m^2$) for m^2 in the usual formalism. Ecker [70], after discussing the lack of Lorentz covariance of these quantization schemes, suggests abandonment of the scalar property of the field operator and therefore of the Lagrangian formalism. Instead, he makes use of methods of axiomatic field theory to construct the Hilbert space and define the fields. The fourth component of his momentum operator is positive definite and therefore again the momentum cannot transform as a four-vector. He also points out the difficulties in describing interactions because of the nonlocalizability of the free tachyon field.

Because of the probable necessity (forced by the problems of the free quantized tachyon field theory) to make all tachyons virtual, the unitarity of a theory incorporating them is not easy to demonstrate (or even to investigate). Boulware [70] has pointed out some of these problems in a type II theory.

Morse and Feshback [53] show that the usual Klein-Gordon equation describes a flexible string embedded in a sheet of rubber which provides a spring constant K. The additional restoring force associated with the rubber makes it possible to eliminate end supports. The equation is

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = \frac{\partial^2\psi}{\partial x^2} - \mu^2\psi$$

where

$c^2 = T/\rho$ and $\mu^2 = \frac{K}{T}$; ψ = displacement of string

T = Tension of the string ; ρ = Linear density of the string

We have found this model helpful in visualizing the situation for tachyons. In this case take K negative. Then the force produced by the rubber is in the direction of the displacement ψ instead of being a restoring force. The instability inherent in the negative mass squared Klein-Gordon equation is then made apparent. Note that such a model is of a type II tachyon theory.

Aharanov, Komar and Susskind [69] have dealt with this instability in a nonlinear model. They look at a system of pendulums coupled by springs to their nearest neighbors. For the case in which the pendulums are in the inverted position, the continuum limit for small displacements yields the negative mass squared equation. They show that the instability occurs when the initial modes involve $k^2 < m^2$. They also show that if these exponentially growing modes are included then there exists a causal Green's function. However, if the unstable modes are excluded then the field cannot couple locally to a source and the Green's function cannot be confined to the interior of the light cone.

The analysis of the propagation of signals by the Klein-Gordon equation with negative mass squared was first made by Ehrenfest in 1910 for a type II theory. Further analysis along this line has indicated that signals will travel at less than c even though

$$v_g \equiv \frac{\partial E}{\partial p} > c$$

See [10] Ehrenfest, [54] Sommerfeld, [69] and [70] Fox, Kuper and Lipson, and [71] Bers, Fox, Kuper and Lipson.

So we can conclude that type II theories may have causal propagation and subluminal signal velocity, but at the price of dynamical instability. Type I theories may avoid the problem of dynamical instability, but for them it is not even possible to define a signal velocity and the propagation is not causal, because of the lack of localizability in such theories.

Although single processes of emission and absorption of tachyons may be adequately treated by means of the reinterpretation principle, causal loops are not so easily disposed of. One example of such a paradox arising from signals exceeding the speed of light, is given by Roger Newton [70]. Two rockets, initially at rest, move with constant velocity for a time and then come to a rest again. One should visualize a Minkowski diagram for the moving and rest systems. The dotted lines indicate simultaneity in each system. See figure (2). According to the usual construction of a Minkowski diagram [23] Lorentz et al, the dotted lines are drawn parallel to the x coordinates for each frame. We indicate one such construction on the figure. Then, we see that a superluminal signal from A to B arrives at a later time as measured in the moving coordinate system. Rocket II at point C then, as a result of receiving the signal, comes to rest and sends another tachyon signal to Rocket I, telling him not to send any further signals. Because of the changing definitions of simultaneity for the systems, this signal arrives before point A, which initiated the whole exchange. Hence the paradox. If the original signal had not been sent, then the return signal would not exist to cause the prevention of the original message. In order

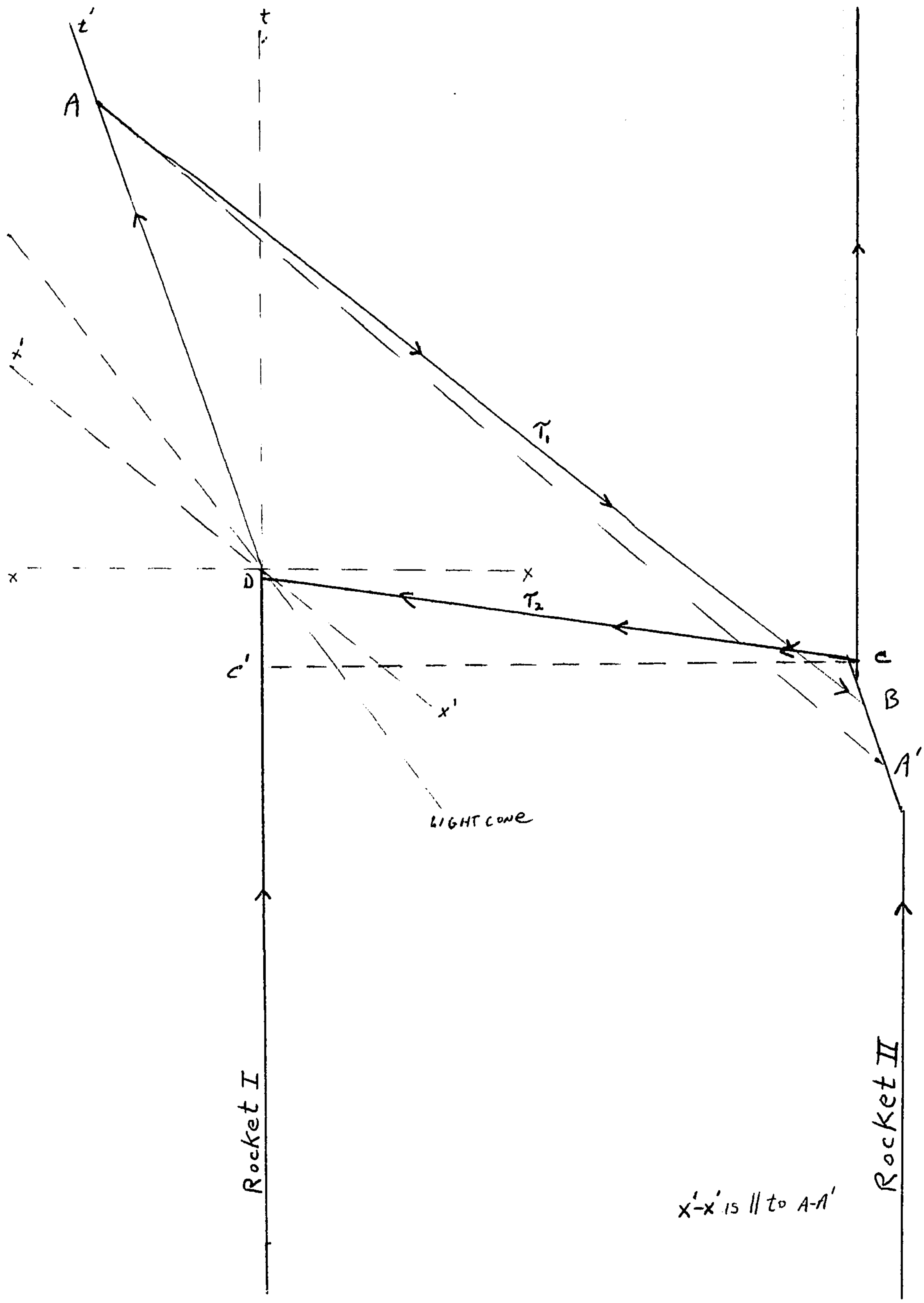


figure 2

to avoid this paradox and the other problems of a free tachyon field theory, we must make an S-matrix theory in which tachyons appear only as virtual internal lines coupled to other quantum particles, and not coupled to sources under the control of an external agency. Then each vertex containing a tachyon is regarded as representing an event in a stochastic process--an event which may be interpreted either as an absorption or an emission depending upon Lorentz frame conventions, but an event which can <u>not</u> be controlled. One cannot decide to send a tachyon from A to B, and so no <u>causality</u> can be ascribed to the events occurring in the Newton loop. It must be admitted that no systematic exposition of the Feynman rules for such a tachyon-containing S-matrix theory has been presented; so it is not easy to decide definitively whether or not such a theory also posseses irreparable flaws. In later sections we attack this problem by considering Feynman diagrams including tachyonic virtual lines with various other lines attached to the tachyonic line, and argue that such attachments (Cherenkov radiation) must be considered and appropriately analyzed if many experimental searches for tachyons are to have any reasonable theoretical basis at all.

Experiments

Motivated by the 1962 article by Bilaniuk et al, Torsten Alväger and Peter Erman [65] at the Nobel Institute in Stockholm began an experimental investigation aimed at finding tachyons. They used radioactive beta decay of Thulium-170 as the possible source of electrically charged superluminal particles. From the relation

$$E^2 = p^2 \pm m^2$$

it is apparent that at equal values of the momentum, tachyons and ordinary particles with equal m will possess different energy. They deflected the products of the decay by means of a magnetic field. Then a particular momentum was selected and its energy measured by the counter in the double focussing beta spectrometer. Although the search continued for a period of two years, from 1963-1965, the sought after difference was not found. It was assumed that there was no Cherenkov Radiation by these charged tachyons.

The first attempt to detect tachyons by means of the expected electromagnetic Cherenkov radiation was reported in 1968 by Alvager and Kreisler ([68] AK, hereafter). In order to enhance the probability of detection, an electric field was used to supply energy to any tachyons created. Thus it was hoped that the time would be extended during which the tachyons emitted radiation at the characteristic angle determined by the velocity according to

$$\cos\theta = c/v_T$$

Arguments were advanced indicating that the energy lost by Cherenkov radiation would equal energy gained from the field at a terminal velocity determined by the tachyon parameters. The tachyons were to be produced by bombardment of lead by gamma rays at the Princeton-Penn. accelerator lab. No tachyons were found, and an upper limit of 3 μb was placed on the photoproduction of tachyons whose charge is from .1 to 2 electron charges.

A second search based on Cherenkov radiation was reported the next year by Davis, Alvager and Kreisler [69] (DAK). The tachyons, to be produced by photons from Co^{60} impinging on lead, were to pass through two of the above detectors with accelerating electric fields. Looking for coincident counts from the two detectors would avoid the large number of spurious counts produced by corona discharge. Again the results were negative.

In addition to possible detectibility by Cherenkov radiation, tachyons have spacelike four momentum and hence, negative mass squared. A combination of two or more tachyons however, can have either spacelike or timelike four momentum because of the possibility of the spatial components of the momentum canceling out. Hence, a missing mass experiment involving missing tachyons might find negative missing mass squared, but if two tachyons are emitted in opposite directions it would be positive. In 1970 Baltay, Feinberg et al reported such a missing mass experiment in which antiprotons or K^- particles were given an opportunity to produce tachyons in a bubble chamber. The experimenters hoped that the experiment would not be sensitive to unproven conjectures about the tachyon interaction with matter. The invariant mass squared of any missing particle is calculated through measurement of the momenta of the observed particle in a bubble chamber by using conservation of energy and momentum. It is assumed that there are no tachyons in the initial state. The missing mass squared was then plotted and a search was made for negative values. Although two or more tachyons can give positive missing mass, no combination of ordinary particles can give a negative missing mass. At first there did appear to be some possibilities for tachyons. But when each of the cases in which negative mass squared was re-examined, some error was found to disqualify it. The authors concluded that the cross-section for production of neutral tachyons was about 1,000 times less than the corresponding cross-section for pions.

In 1971 Danburg et al published their findings on a search for charged tachyons. They sought bubble chamber tracks of tachyon pairs produced in the reaction:

$$K^- + p \rightarrow \Lambda + t^+ + t^-$$

Although it was assumed that the charged particles would leave tracks, it was also postulated that there would be no Cherenkov radiation. In addition, it was assumed that the tachyons follow curved paths in the magnetic field just as ordinary particles do. We will later compare this assumption with our analysis of tachyon trajectories. Again no negative mass squared candidates were found.

In the spirit of the Generalized Lorentz Transformation, ([74] Mignani & Recami) and a postulated symmetry between superluminal and ordinary worlds, it has been suggested that tachyons might possess magnetic charge ([69] Parker). An attempt to find these monopoles has been made by Bartlett and Lahana [72]. A longitudinal magnetic field is used to accelerate the monopole to a terminal velocity and the expected Cherenkov radiation is sought.

The 1-MeV gamma rays from a Co^{60} source did not produce any detectable tachyon monopoles through photoproduction. The authors set upper limits of about 10^{-36} cm^2 for the cross sections in lead and water.

Danburg and Kalbfleisch [72] looked for instances in which protons in a bubble chamber at Brookhaven National Laboratory participated in the reaction $p \rightarrow p + T$. This is kinematically allowed for a moving proton. They looked for events in which there were no incident particles, and yet the proton suddenly moved (recoiled) for no apparent reason. Although they found examples of this, on further analysis they could all be explained in more mundane terms.

The most obvious property of tachyons, namely their great speed, which always exceeds that of ordinary particles, has also been exploited. Any tachyons which are associated with the extensive air showers (EAS) created by high energy cosmic rays in the atmosphere, would arrive before the slower constituents. The fastest ordinary particles are those with the highest energy and have velocities virtually equal to c. The fastest tachyons will be those with the lowest energy, and would arrive almost instantaneously. By triggering the counter on any signal and then looking at the record for up to 60 micro-seconds afterwards, one seeks a correlation indicating that the first signal was the precursor of a shower of ordinary particles.

The first such experiment was carried out by Ramana Murthy [71] at the Tata Institute in India. The time interval considered was twenty microseconds. The only coincidences found did not exceed the numbers expected due to chance.

The only positive result so far is that of Clay and Crouch reported in Nature in March, 1974. They studied EAS which were two orders of magnitude greater than previously reported on by Raman Murthy. The cosmic ray showers were of energy approximately 2×10^{15} eV. A total of 1,307 showers were analyzed. The results show that the distribution of large pulses following a triggering is not uniform. A χ^2 test indicates only one chance out of one hundred that the data is from a uniformly distributed population.

Although there is a possibility of the correlation arising from other sources, the authors indicate that this seems unlikely. The production of associated particles at the source would have to be followed by continuing association throggh the interstellar and source magnetic fields, which seems doubtful.

A subsequent study of cosmic ray extensive air showers in Japan has not found evidence of tachyons ([74] Tanahashi & M.F.Crouch).

Chapter III

The Dynamics of a Tachyonic Source

In the absence of a coherent quantum field theory or quantum S matrix theory of interacting tachyons, we investigate the properties to be expected from "reasonable" assumptions. If such particles are to be experimentally detectable, some idea of their behaviour should be helpful in designing a successful experiment. In fact, for particles whose behaviour promises to be so novel, this information is much more important than is ordinarily the case.

In this Chapter we extend our consideration of those aspects of superluminal particles associated with Cherenkov radiation. This Chapter may be thought of as an investigation of Cherenkov radiation with an emphasis on the behavior of the source. The next Chapter will emphasize the behavior of the radiated field. In each case the quantum and classical aspects will be contrasted and compared

As noted in Chapter II, BDS have pointed out many tachyonic properties which follow from having a negative mass squared and from employing the usual Lorentz covariant equations. In particular they look at the implications of:

$$E^2 = p^2c^2 - m_\tau^2 c^4 \qquad (1)$$

where m_τ is real. In this thesis we will only use a real mass parameter. If $\bar{v} \equiv \bar{p}c^2/E$, then

$$E = mc^2/\sqrt{v^2/c^2-1} \qquad (2)$$

and the energy of the tachyon decreases as the velocity increases. As $v \to \infty$ we see that $E \to 0$. In other words, if you push on the tachyon in the direction of its motion it will gain energy and slow down. The harder you push, the more it will slow down. Furthermore, if the tachyon is losing energy through Cherenkov radiation it will speed up.

Although this speeding up follows qualitatively from equation (2), kt would be illuminating to see quantitatively how the force and acceleration can be in opposite directions.

We derive an equation for the acceleration of a classical particle under the influence of a prescribed force. The equation is valid for both superluminal and ordinary particles. We make no assumption about the nature of the force. In particular examples the force may be produced by an electromagnetic field such as that in one of the experiments already conducted, or may be produced in the reaction associated wkth Cherenkov emission of electromagnetic radiation or of a massive radiation.

We have the well known relation for the 3-velocity $\bar{v}$ in terms of the three momentum $\bar{p}$ and the energy E:

$$\frac{d\bar{x}}{dt} = \bar{v} = \frac{\bar{p}c^2}{E} \tag{3}$$

Taking derivatives of (3) with respect to time:

$$\frac{d\bar{v}}{dt} = \frac{d}{dt}\left(\frac{\bar{p}}{E}c^2\right) = \frac{d\bar{p}}{dt}\frac{c^2}{E} - \frac{\bar{p}c^2}{E^2}\frac{dE}{dt} \tag{4}$$

Now use

$$\frac{dE}{dt} = \bar{F}\cdot\bar{v} \quad \text{and} \quad \frac{d\bar{p}}{dt} = \bar{F} \tag{5}$$

as usual. Combining (5) and (4):

$$\frac{d\bar{v}}{dt} = \bar{F}\frac{c^2}{E} - \bar{p}\frac{c^2}{E^2}\bar{F}\cdot\bar{v} \tag{6}$$

Combining terms and using a component natation equation (7) becomes:

$$\frac{dv_i}{dt} = \frac{c^2}{E} F_j \left(\delta_{ji} - v_j p_i/_E\right) \tag{7}$$

or

$$\frac{d\bar{v}}{dt} = \frac{c^2}{E}\bar{F}\cdot\left(\bar{\bar{I}} - \frac{\bar{v}\bar{p}}{E}\right)$$

Where the outer product $\bar{A}\,\bar{B}$ is an operator on vectors defined by $(\overline{\overline{AB}})\bar{C} = \bar{A}(\bar{B}\cdot\bar{C})$. Now use the equation (3) for the second term:

$$\frac{dv_i}{dt} = \frac{c^2 F_j}{E}\left(\delta_{ji} - \frac{v_j v_i}{c^2}\right) \tag{8}$$

or

$$\boxed{\frac{d\bar{v}}{dt} = \frac{c^2\bar{F}}{E}\cdot\left(\bar{\bar{I}} - \frac{\bar{v}\bar{v}}{c^2}\right)} \tag{9}$$

This equation is valid for any finite velocity. If $v < c$ the acceleration is dominated by the first term which is in the direction of the force. If $\vec{F}$ is perpendicular to $\bar{v}$, abbreviated $F \perp \bar{v}$, (the transverse case), or if $\bar{F}$ is parallel to $\bar{v}$, abbreviated $F \parallel \bar{v}$ (the longitudinal case) (9) leads to the definition of the transverse and longitudinal mass parameters:

$$\frac{|\bar{F}|}{|\bar{a}|} = m_t = \frac{m_0}{\sqrt{1 - v^2/c^2}} \quad , \quad \bar{F} \perp \bar{v} \tag{10a}$$

$$m_\ell = \frac{m_0}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}} \quad , \quad \bar{F} \parallel \bar{v} \tag{10b}$$

We can generalize this definition of directional mass and define the symmetric mass matrix:

$$\left(m^{-1}\right)_{ij} \equiv \frac{c^2}{E}\left(\delta_{ij} - \frac{v_i v_j}{c^2}\right) \tag{11}$$

To set the stage for the superluminal case we recall that the relativistic correction which gives rise to a component of acceleration not collinear with the force is responsible for 7 seconds of arc per century of the precession of the perihelion of Mercury (1/6 of the total non-Newtonian precession). As is well known there are corrections to the electron orbits in atoms due to this same cause. The point is that the tachyonic effect to be discussed is not completely unprecedented. As v approaches c the noncollinear component becomes much more important.

For the case $v > c$ we consider first the transverse and longitudinal cases.

a) $\bar{F} \perp \bar{v}$ - In this case $\bar{F} \cdot \bar{v} = 0$ and equation (9) shows us that an attractive force or repulsive force cause the usual acceleration. Hence a tachyon can have circular orbits in any attractive central force field.

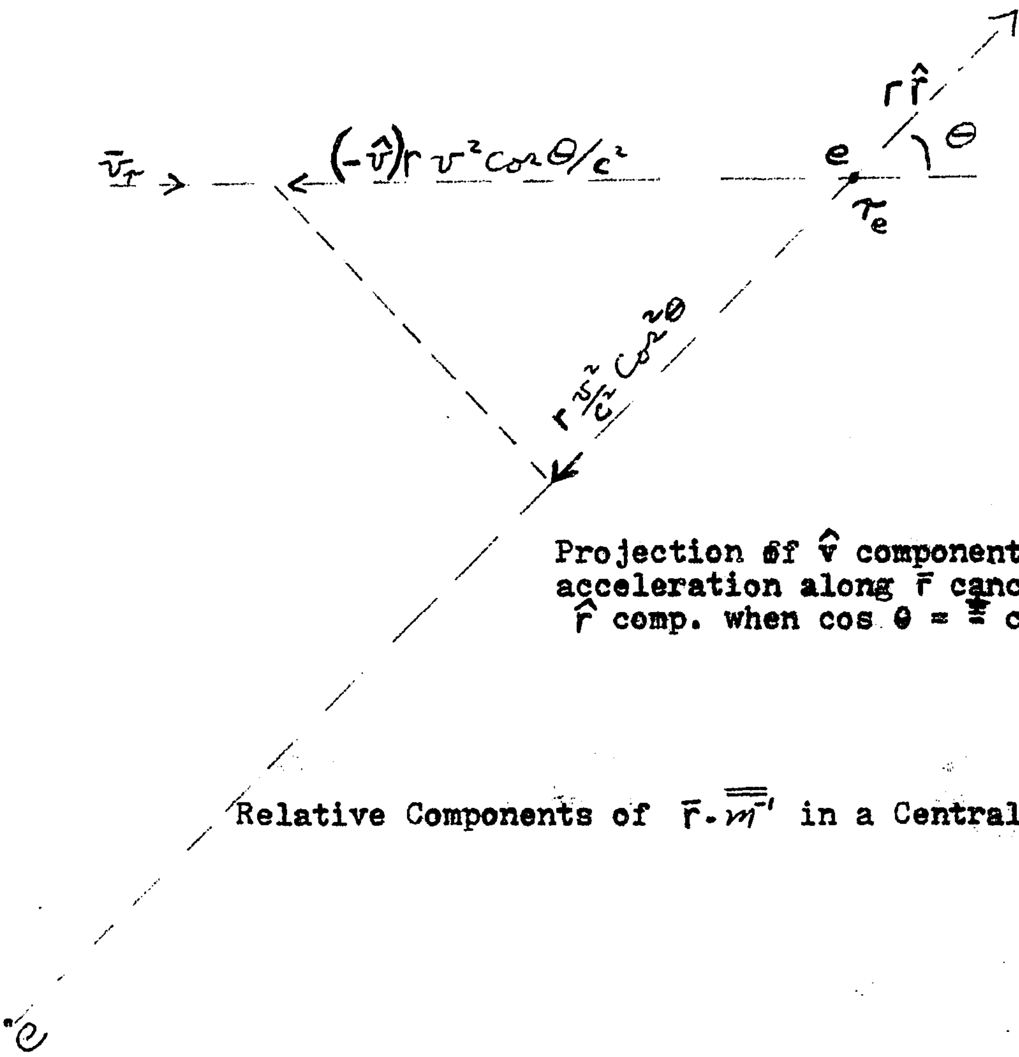
b) $\bar{F} \parallel \bar{v}$ - In this case, equation (9) and (11) show us that the effective inertial mass is negative. The force and acceleration are in opposite directions. A potential which is attractive for transverse motions will be repulsive for longitudinal motions, and vice versa.

For arbitrary angles between $\bar{F}$ and $\bar{v}$ Glück [70] has calculated orbits for a charged tachyon moving about an electric charge under the condition that it is arbitrarily prohibited from emitting Cherenkov radiation. He concludes that circular orbits are the only bound solution. In appendix (H) we calculate the trajectory of a classical non-radiating tachyon moving in a constant electric or magnetic field. But we remark that there appears to be little justification for forbidding Cherenkov radiation; the analogous prohibition for tardyons would forbid electromagnetic self-energy corrections (and thus drastically alter the Lamb shift).

We assert that the calculation of any trajectory of a charged tachyon in an electromagnetic field which does not include the effects of the radiation reaction force associated with Cherenkov radiation is inconsistent, and probably a poor approximation.

It is apparent that equation (8) will usually give a component of acceleration along $\bar{F}$ (or $\bar{r}$ for a central force) and a component along $\bar{v}$. At some point on a trajectory the central part of the acceleration will change from attractive to repulsive (or vice versa) as a tachyon leaves the vicinity of a static charge. Figure (3) shows the relative magnitude of the two components of acceleration of a tachyon $\mathcal{T}$ moving in a static Coulomb field produced by a charge of like sign. The full acceleration is, from (9)

$$\frac{d\bar{v}}{dt} = \frac{c^2 e^2}{r^3}\,\bar{r}\cdot\left(\bar{\bar{I}} - \frac{\bar{v}\bar{v}}{c^2}\right) \tag{12}$$



Projection of $\hat{v}$ component of acceleration along $\bar{r}$ cancels $\hat{r}$ comp. when $\cos\theta = \mp c/v_T$

Relative Components of $\bar{r}\cdot\overline{\overline{m}}^{-1}$ in a Central Potential

ACCELERATION OF TACHYON

Figure 3

The acceleration in the radial direction is seen to change sign when (see construction in figure (3))

$$\cos \theta = \pm c/v_{\gamma} \tag{13}$$

The solution using the plus sign represents the usual Cherenkov radiation angle. We see that when the static source lies within the Cherenkov cone of the tachyon, the tachyon experiences an acceleration towards the static charge instead of the naively anticipated repulsion.[3]

We next turn to a consideration of the Lorentz transformation properties of the kinematical and dynamical quantities associated with the classical motion of a point particle along a tachyonic trajectory. Consider two neighboring points on such a world line, with coordinates (x,y,z,t) and ($x+\Delta x$, $y+\Delta y$, $z+\Delta z$, $t+\Delta t$).

To focus attention on the most interesting aspects of the situation, let us confine ourselves to the case in which y=z=0 for all points on the trajectory, and let us consider only those Lorentz transformations which maintain this condition ($y'=z'=0$), that is, consider only Lorentz boosts.

The usual boost transformation in the x-t plane when applied to (Δt, 0, 0, Δx) gives

$$\Delta x' = (\Delta x - u\Delta t)\gamma_u \quad ; \quad \gamma_u \equiv \frac{1}{\sqrt{1-u^2/c^2}} \tag{14}$$

or

$$\Delta x' = \Delta x(1 - u/v_T)\gamma_u \quad , \text{ with } v_T \equiv \frac{\Delta x}{\Delta t} \tag{15}$$

Assuming that the tachyon velocity $v_T>c$, we see that there is no Lorentz boost transformation which changes the sign of Δx since $u<c$. For ordinary particles a change of sign of Δx occurs when $u>v$ and gives rise to a reversal in the direction of the velocity.

The Lorentz boost transformation of Δt is:

$$\Delta t' = \left(\Delta t - \frac{\bar{u}\cdot\Delta\bar{x}}{c^2}\right)\gamma_u \tag{16}$$

$$= \Delta\left(1 - \frac{\bar{u}\cdot\bar{v}_T}{c^2}\right)\gamma_u \tag{17}$$

Now a transformation velocity $\bar{u}$ such that

$$\frac{\bar{u} \cdot \bar{v}_T}{c^2} > 1 \tag{18}$$

will change the sign of t. It will also change the sign of the fourth component all of four-vectors tangent to the tachyonic world-line, in particular, the energy-momentum four-vector (BDS, [67] Feinberg). Note that the invariance of the sign of (15) also applies to the space components of all four-vectors tangent to the tachyonic world line. For example, a transformation satisfying (18) and changing the sign of Δt and E will not change the sign of the momentum p_x.

The tachyon velocity changes sign because of the sign change of Δt, not because of the sign change of Δx; an ordinary particle experiences velocity reversal under appropriate boosts for the "opposite" reason. After the boost the energy is not only negative, but the velocity and momentum point in opposite directions.

We see then that the "reinterpretation principle" will get the velocity and momentum in the same direction.[4]

We wish to consider a tachyonic field which asymptotically moves freely. Assume it is localizable in the sense that it should be possible to make wave packets which move along classical point tachyon trajectories. For the moment we ignore the difficulties in obtaining such localizability in order to investigate some consequences that would follow from the existence of such states. The matrix elements of the current operator associated with such localizable states should yield a four-vector $(\rho, J, 0, 0)$ proportional to the tachyonic tangent vector $(\Delta t, \Delta x, 0, 0)$.

But:

$$\rho' = (\rho - \bar{J}\cdot\bar{u})\gamma_u \tag{19}$$

$$\bar{J}' = (\bar{J} - \rho\frac{\bar{u}}{c^2})\gamma_u \tag{20}$$

we expect that

$$J = \rho v_T$$

so

$$\mathrm{sign}[\rho'] = -\mathrm{sign}[\rho] \quad \text{for } \frac{\bar{u}\cdot\bar{v}_T}{c^2} > 1$$

In this case the tachyon is conventionally reinterpreted as an antitachyon of opposite charge, travelling in the direction opposite to the initial velocity.

If charge is to be conserved globally we must either consider simultaneously the source and sink of the tachyon or the flow through the boundary of the volume we are considering. In figure (4) we consider one of the problems encountered when one attempts to consider an asymptotically free charged tachyon. We see that the reversal of sign of the tachyon charge density under a Lorentz boost transformation does not destroy the scalar nature of the total charge $\int d^3x\,\rho$, when the tachyon appears in an intermediate state in figure (4a). However $\int d^3x\,\rho$ is not Lorentz invariant when the tachyon is asymptotically free, figure (4b). This is because too much charge flows through the spatial boundaries at infinity. This is also seen by considering the charge:

$$i\int \phi^* \overleftrightarrow{\partial_0}\, \phi\, d^3x$$

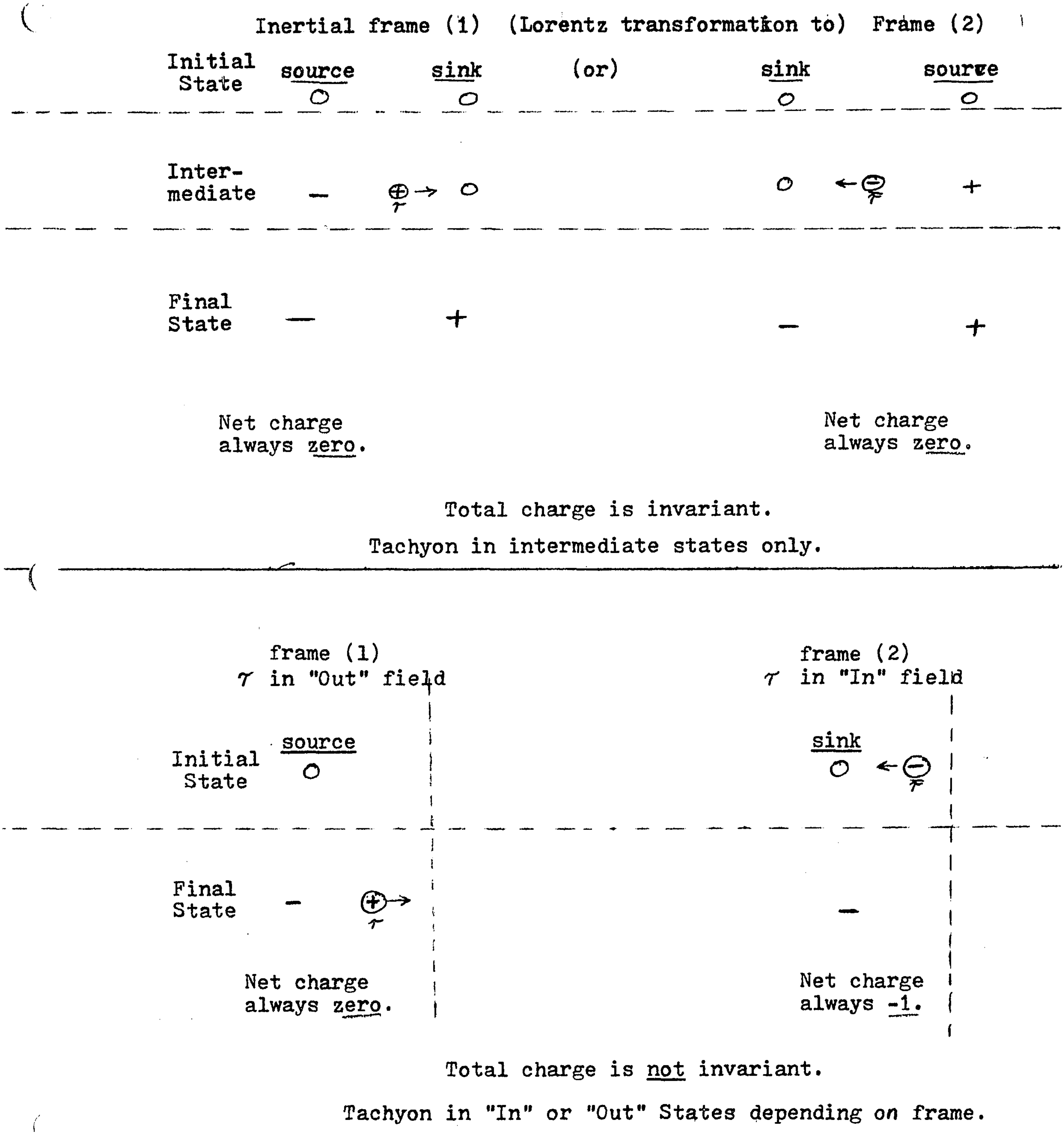

Inertial frame (1) (Lorentz transformation to) Frame (2)
Initial State
source
sink
(or)
sink
source
Inter-mediate
Final State
Net charge always zero.
Net charge always zero.
Total charge is invariant.
Tachyon in intermediate states only.
frame (1)
τ in "Out" field
frame (2)
τ in "In" field
Initial State
source
sink
Final State
Net charge always zero.
Net charge always -1.
Total charge is not invariant.
Tachyon in "In" or "Out" States depending on frame.

Figure 4.

resulting from a free tachyon wave packet:

$$\phi(\bar{r},t) = \int d^4k \; \delta(-k^2+m^2)\, F(\bar{k},\omega)\, e^{ikx}$$

we find:

$$Q = \int_{-\infty}^{\infty} d\omega \; \epsilon(\omega) \; K(\omega)\, d\Omega \, |F(\hat{k},\omega)|^2$$

where

$$K(\omega) \equiv \sqrt{\omega^2+m^2} \quad , \quad \epsilon(\omega) \equiv \frac{\omega}{|\omega|}$$

and $d\Omega$ is the element of solid angle.

The reversal of the charge is manifested by the sign function (not invariant for tachyons) $\epsilon(\omega)$ within the integral. The usual proof of the invariance of Q relies on both current conservation , $\partial_\mu j^\mu = 0$, and discarding an integral at spatial infinity. We see that the former is still true while the latter is no longer justified.

We note that when $v_T = \infty$ and $E_T = 0$ the tachyon is a pure current, i.e. $\rho = 0$ and $\bar{J}$ is finite, [(19) and (20)].

The velocity of both subluminal and superluminal particles can change direction under certain Lorentz boosts. The situation is completely different for the acceleration three-vector. Consider a tachyon emitting Cherenkov radiation and speeding up as it loses energy. In a boosted inertial frame in which the velocity is reversed, the direction of "speeding up" will be along the new velocity if the rule that Cherenkov radiation reaction causes an increase of speed is Lorentz invariant, as we expect it to be. Therefore, the acceleration is reversed. For ordinary particles, the acceleration does not reverse under Lorentz boost transformations in the appropriate x-t plane, (it clearly does reverse under rotational Lorentz transformations).

To show that this picture of Lorentz boosted acceleration reversal (derived on the basis of assumptions concerning Cherenkov effects) is correct, we derive the transform of the acceleration directly. For simplicity we continue to consider $\bar{u}$ to be along $\bar{v}_r$. First, divide equation (15) by equation (17) to obtain the usual collinear velocity addition equation:

$$\bar{v}' = \frac{\bar{v} - \bar{u}}{1 - \frac{\bar{u}\cdot\bar{v}}{c^2}} \tag{21}$$

Take derivatives of both sides with respect to t.

$$\frac{dv'}{dt} = \frac{\frac{dv}{dt}\left(1 - \frac{uv}{c^2}\right) + \frac{u}{c^2}\frac{dv}{dt}(v-u)}{\left(1 - \frac{uv}{c^2}\right)^2} \tag{22}$$

$$= \frac{dv}{dt}\frac{\left(1 - \frac{uv}{c^2} + \frac{uv}{c^2} - \frac{u^2}{c^2}\right)}{\left(1 - \frac{uv}{c^2}\right)^2} \tag{23}$$

Now use (17) to replace dt on the left hand side and obtain:

$$\frac{dv'}{dt'} = \frac{dv}{dt}\frac{\left(1 - \frac{u^2}{c^2}\right)^{3/2}}{\left(1 - \frac{u\cdot v_r}{c^2}\right)^3} \tag{24}$$

Thus we see directly from this transformation formula for acceleration that the condition for reversal, (18), is the same as for the sign change of E, Δt and $\bar{v}_r$. We also see that for ordinary particles where $v < c$, the acceleration never changes sign under such boosts. Note that the acceleration of a nonradiating tachyon in a constant electric field (described in appendix H) has the Lorentz transformation properties just derived.

Our next step is to analyze the equation obtained by applying conservation of energy and momentum to the process of Cherenkov radiation. Our analysis will be patterned after that given by Einstein for ordinary radiation ([17]Einstein). We assume that there is no interference of successive emissions. The calculation is inherently quantum mechanical, i.e. the energy and momentum are radiated in discrete quanta. We take the radiation process to be stochastic; there will be a probability $W(p_\mu^\tau, p_\mu^\pi)$ of emission during an interval dt by a tachyon with initial four-momentum p_μ^τ of a π-quantum having momentum four-vector p_μ^π. The probability per unit time $W(p_\mu^\tau, p_\mu^\pi)$ will be independent of the previous history of the tachyonic source, but it may depend on the density of π-quanta in the vicinity of the tachyon (possibility of induced emission). We shall require that W transform appropriately under change of inertial reference frame so that the Cherenkov process will be invariantly described. There will be a finite recoil of the tachyon with each emission, a recoil we compute using conservation of four-momentum and the condition that all particles involved maintain the appropriate mass-shell condition. The first result will be a determination of the angles and energies with which the quanta are emitted. This calculation is complementary to that in the next section, which gives the <u>average</u> recoil rate of the tachyon as a function of its velocity.

In this section we derive the main result for the

Cherenkov emission of π-particles having a positive mass-squared. We consider more general kinematic situations in appendix A.

Conservation of energy and momentum during the emission of a π particle by a tachyon τ says that:

$$p_\mu^\tau = p_\mu^\pi + p_\mu^{\tau'} \tag{25}$$

where $p_\mu^{\tau'}$ is the four-momentum of the tachyon after the emission process is completed. Therefore:

$$\left(p_\mu^\tau - p_\mu^\pi\right)^2 = \left(p_\mu^{\tau'}\right)^2 \tag{26}$$

and

$$p_\mu^\tau \cdot p_\mu^\tau - 2\, p_\mu^\tau p_\mu^\pi + p_\mu^\pi p_\mu^\pi = p_\mu^{\tau'} p_\mu^{\tau'} \tag{27}$$

Now using the facts that the tachyon and the π are on the mass shell before and after the emission process:

$$p_\mu^\tau p_\mu^\tau = p_\mu^{\tau'} p_\mu^{\tau'} = -m_\tau^2 \tag{28}$$

and

$$p_\mu^\pi p_\mu^\pi = m_\pi^2 \tag{29}$$

in (27).

Hence:

$$p_\mu^\tau p_\mu^\pi = \frac{m_\pi^2}{2} \tag{30}$$

Now write out the left hand side

$$E_\tau E_\pi - \bar{p}_\tau \cdot \bar{p}_\pi = \frac{m_\pi^2}{2} \tag{31}$$

and divide by $|\bar{p}_T||\bar{p}_\pi|$ and use the relation $v = \frac{pc^2}{E}$. We obtain the angle θ at which the massive π particle is emitted as a function of the π parameters:

$$\cos\theta = \frac{c^2}{v_T v_\pi} - \frac{m_\pi^2}{2|\bar{p}_T||\bar{p}_\pi|} \tag{32}$$

For the case $m_\pi = 0$ (which implies $v_\pi = c$) this relation yields the usual Cherenkov angle for electromagnetic radiation:

$$\cos\theta = \frac{c}{v_T}$$

Note that for the case $m_\pi \neq 0$ there is a range of values of θ for any given value of v_T. We analyze equation (32) in detail in the next Chapter. Also, in appendix A, we derive a generalization of (32) for the case in which the internal state of the tachyon is allowed to change.

Note that for electromagnetic Cherenkov radiation in a medium we have:

$$p_\mu^\pi \rightarrow p_\mu^\gamma = \hbar\left(\omega, \frac{n\omega}{c}\right) \quad \text{and} \quad n > 1$$

$$m_\pi^2 \rightarrow p_\mu^\gamma p_\mu^\gamma = \hbar^2\omega^2(1-n^2) < 0$$

i.e. "spacelike"

Then equation (32) becomes:

$$\cos\theta = \frac{c^2}{v_T n} + \frac{\hbar n\omega}{2c|\bar{p}_T|}\left(1 - \frac{1}{n^2}\right) \tag{33}$$

This agrees with the usual quantum correction to massless Cherenkov radiation ([58] Jelley). Note that the correction term disappears as $n \rightarrow 1$ or $\hbar \rightarrow 0$.

We now turn to a study of the average rate at which a tachyonic source obeying our stochastic model will lose energy through the emission of Cherenkov radiation. We note first that a tachyon travelling in a vacuum and emitting Cherenkov radiation possesses no preferred reference frame, except possibly one in which $v_\tau = \infty$. Whatever the form of the coupling to the field which is Cherenkov radiated and whether or not this field is massive or massless, Lorentz covariance imposes restrictions on the rate of energy loss and on the equations of motion.

We assume that during Cherenkov radiation the "internal" state of the tachyon does not change. As it radiates and loses kinetic energy, the tachyon speeds up. However, a Lorentz boost transformation in the direction of the instantaneous tachyon velocity v_τ, will also change the tachyon velocity. Let the average energy radiated during the time interval dt, with the original tachyon velocity $\bar{v}$, be given by $dE = \Gamma(\bar{v})dt$. As time progresses $\dot{v}$ will increase as a result of the energy loss; and its direction will change as a result of the recoil calculated in the last section. Therefore $\Gamma(\bar{v}(t))$ will be a function of time. We have assumed that $\Gamma(\bar{v})$ depends not only on the magnitude but also on the direction of $\bar{v}_\tau$. Because of the constantly changing tachyon direction the possibility exists that the tachyon may absorb a previously emitted π or be induced to emit further π's by it. We consider the probability of this to be negligible.

We assume that the dependence of $\Gamma(\bar{v})$ on $|\bar{v}|$ can be deduced from the Lorentz covariance of the description of the Cherenkov radiation process.

The usual form for the energy loss of a particle emitting electromagnetic radiation via the Cherenkov effect is ([58] Jelley, [62] Jackson):

$$\frac{dE}{dx} = -\frac{e^2}{c^2}\int \omega\left[1-\frac{1}{\beta^2 n^2(\omega)}\right]d\omega \qquad (34)$$

where

$\beta = v/c$, n = index of refraction

In a medium where a fast electron exceeds the phase velocity of light the range of integration is over those frequencies such that $\beta^2 n^2(\omega) > 1$. Alvåger and Kreisler ([68] AK), Bartlett and Lahana [72], and Davis, Alvåger and Kreisler ([69] DAK) based their searches for tachyons on this equation with $E_T/\hbar$ as the cutoff or upper limit for the integration. This equation is inadequate for tachyons in a vacuum for two reasons:

a) dE/dx is not a Lorentz covariant function of $\bar{v}$.

b) a positive range of emergies for the tachyon will contain negative energies after a Lorentz transformation, invalidating the rationale behind the frequency cutoff. In fact, Lorentz covariance and dimensional considerations will make it apparent that the tachyon theory must provide an additional parameter, e.g. m_T or a k_{max}, to serve as a scaling factor or cutoff in order for the Cherenkov radiation to be finite.

We now explore our averaged stochastic model in quantitative detail. The tachyon τ is assumed to travel a certain distance Δr, on the average between successive emissions of π-particles. Δr begins after the last π emission and ends after the next one. Δr can depend on the speed of the tachyon. We suppose that the tachyon emits, on the average, a quantity ΔE of energy with each π-emission. ΔE_τ can depend on v_τ, as can the average time between emissions, satisfying $\Delta r = v_\tau \Delta t$. The direction of $\Delta \bar{r}$ will be parallel to the instantaneous ve[illegible]hyon.

Consider a s[illegible]path $\Delta \bar{r}$ in figure (5). The tachyon four[illegible] this segment is denoted $p_\tau^{(1)}$ [illegible]m $p_\tau^{(2)}$ exists after the next [illegible] these four momenta with the points ma[illegible]gure (5). $\Delta E = E_\tau(2) - E_\tau(1)$ i.e. the energy loss [illegible] Δr is the loss at the second emission.

Now look at this same segment of the tachyon path in a frame boosted by velocity $\bar{u}$ which is parallel to $\Delta \bar{r}$ and therefore to $\bar{v}_\tau$. We have:

$$E'_\tau(1) = \left(E_\tau(1) - \bar{p}_\tau(1) \cdot \bar{u}\right)\gamma_u \tag{35}$$

$$E'_\tau(2) = \left(E_\tau(2) - \bar{p}_\tau(2) \cdot \bar{u}\right)\gamma_u \tag{36}$$

Subtracting (35) from (36)

$$\Delta E'_\tau = E'_\tau(2) - E'_\tau(1) = \left(\Delta E_\tau - \left(\bar{p}_\tau(2) - \bar{p}_\tau(1)\right) \cdot \bar{u}\right)\gamma_u \tag{37}$$

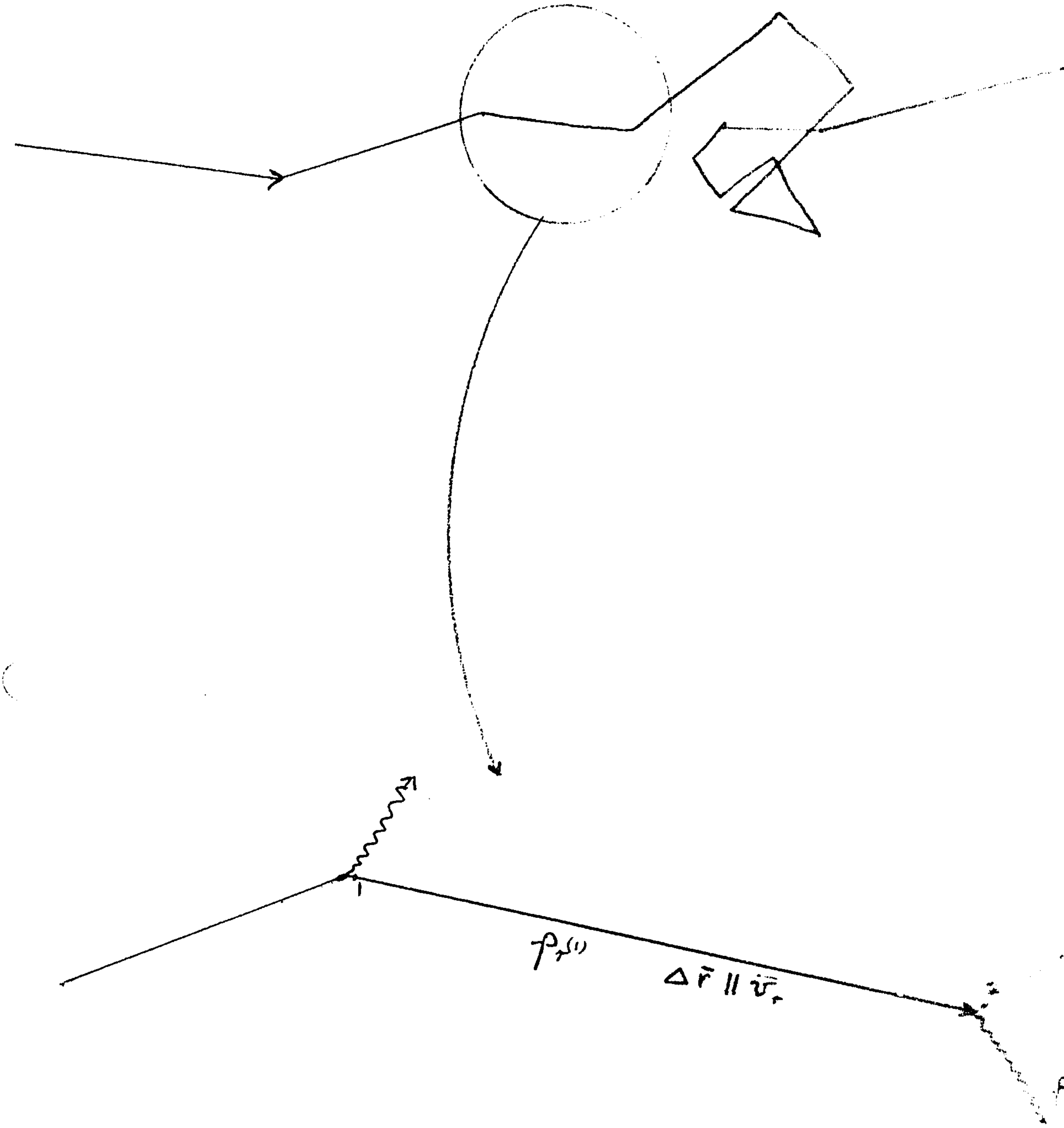


Figure 5

We now explore our averaged stochastic model in quantitative detail. The tachyon τ is assumed to travel a certain distance Δr, on the average between successive emissions of π-particles. Δr begins after the last π emission and ends after the next one. Δr can depend on the speed of the tachyon. We suppose that the tachyon emits, on the average, a quantity ΔE of energy with each π-emission. ΔE_τ can depend on v_τ, as can the average time between emissions, satisfying $\Delta r = v_\tau \Delta t$. The direction of $\Delta \bar{r}$ will be parallel to the instantaneous velocity of the tachyon.

Consider a segment of the tachyon path $\Delta \bar{r}$ in figure (5). The tachyon fonr-momentum associated with this segment is denoted $p_\tau^\mu(1)$. The tachyon four-momentum $p_\tau^\mu(2)$ exists after the $n p_\tau^\mu($ π-emission. We associate these four momenta with the points marked (1) and (2) in figure (5). $\Delta E = E_\tau(2) - E_\tau(1)$ i.e. the energy loss associated with Δr is the loss at the second emission.

Now look at this same segment of the tachyon path in a frame boosted by velocity $\bar{u}$ which is parallel to $\Delta \bar{r}$ and therefore to $\bar{v}_\tau$. We have:

$$E_\tau'(1) = \left(E_\tau(1) - \bar{p}_\tau(1) \cdot \bar{u}\right)\gamma_u \tag{35}$$

$$E_\tau'(2) = \left(E_\tau(2) - \bar{p}_\tau(2) \cdot \bar{u}\right)\gamma_u \tag{36}$$

Subtracting (35) from (36)

$$\Delta E_\tau' = E_\tau'(2) - E_\tau'(1) = \left(\Delta E_\tau - \left(\bar{p}_\tau(2) - \bar{p}_\tau(1)\right) \cdot \bar{u}\right)\gamma_u \tag{37}$$

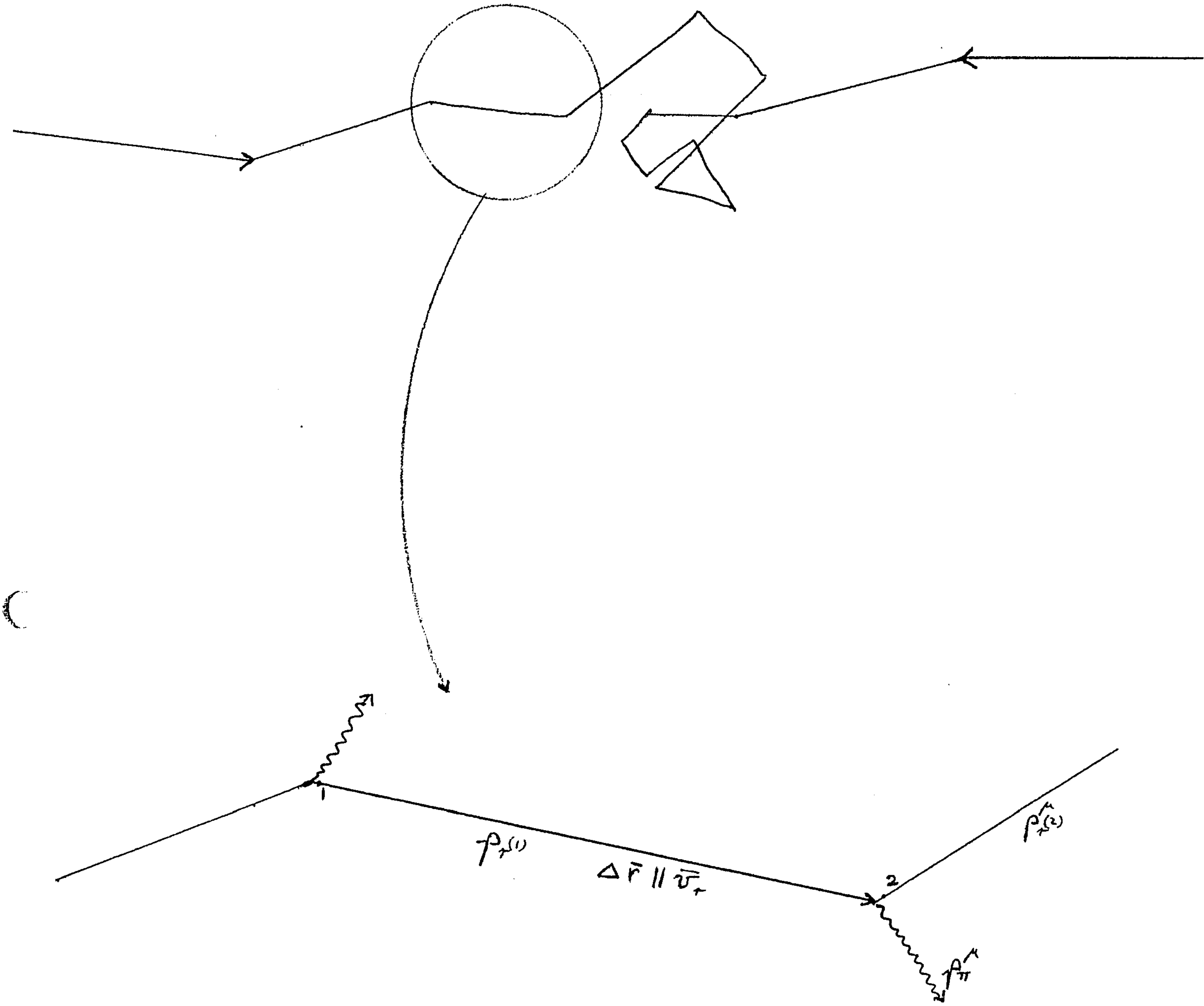


figure 5

By conservation of four-momentum

$$-\left(\bar{p}_\tau^{(2)} - \bar{p}_\tau^{(1)}\right) = \bar{p}_\pi \tag{38}$$

Hence

$$\Delta E_\tau' = \left(\Delta E_\tau + \bar{p}_\pi \cdot \bar{u}\right)\gamma_u \tag{39}$$

Now by definition $\bar{u}$ is parallel to $\bar{p}_\tau(1)$ so we can write:

$$\bar{p}_\pi \cdot \bar{u} = \bar{p}_\pi \cdot \bar{p}_\tau^{(1)} \frac{|\bar{u}|}{|\bar{p}_\tau^{(1)}|}$$

Using equation (31) for $\bar{p}_\pi \cdot \bar{p}_\tau^{(1)}$

$$\bar{p}_\pi \cdot \bar{u} = \left(E_\tau E_\pi - \frac{m_\pi^2}{2}\right)\frac{|\bar{u}|}{|\bar{p}_\tau|}$$

and now note that $E_\pi = -\Delta E_\tau$ and $\frac{E_\tau}{|\bar{p}_\tau|} = \frac{1}{v_\tau}$

Therefore

$$\bar{p}_\pi \cdot \bar{u} = -\frac{\Delta E_\tau |\bar{u}|}{v_\tau} - \frac{m_\pi^2 |\bar{u}|}{2|\bar{p}_\tau|} \tag{40}$$

Then (39) becomes

$$\Delta E_\tau' = \Delta E_\tau \left(1 - \frac{|\bar{u}|}{v_\tau} - \frac{m_\pi^2 |\bar{u}|}{2\Delta E_\tau |\bar{p}_\tau|}\right) \tag{41}$$

For electromagnetic Cherenkov radiation where $m_\pi = 0$ we have:

$$\Delta E_\tau' = \Delta E_\tau \left(1 - \frac{|\bar{u}|}{v_\tau}\right)\gamma_u$$

This will also be a good approximation for nonzero m_π when the last term in (41) can be neglected, i.e. when:

$$v_\tau << \frac{2\Delta E_\tau |\bar{p}_\tau|}{m_\pi^2}$$

This can also be written

$$E_\tau \Delta E_\tau >> \frac{m_\pi^2}{2}$$

For high enough tachyon velocity where $E_\tau \rightarrow 0$ this cannot be satisfied. However, in a frame in which the tachyon energy is great enough it will be satisfied since ΔE_τ is certainly greater than m_π.

Assuming therefore that either we are dealing with electromagnetic Cherenkov radiation or the tachyon energy is sufficiently great, we have:

$$\Delta E_\tau' = \Delta E_\tau \left(1 - \frac{u}{v_\tau}\right) \gamma_u \tag{42}$$

The Lorentz transform of the time interval in which the energy is radiated is:

$$\Delta t' = \left(\Delta t - \frac{\Delta \bar{r} \cdot \bar{u}}{c^2}\right) \gamma_u \tag{43}$$

$$= \Delta t \left(1 - \frac{\bar{v}_\tau \cdot \bar{u}}{c^2}\right) \gamma_u$$

Hence combinmng (42) and (43)

$$\frac{\Delta E_\tau'}{\Delta t'} = \frac{\Delta E_\tau}{\Delta t} \frac{\left(1 - \frac{u}{v_\tau}\right) \gamma_u}{\left(1 - \frac{u \cdot v_\tau}{c^2}\right) \gamma_u}$$

Now $\bar{u} \parallel \bar{v}_\tau$ by definition, and using equation (21) for the composition of collinear velocities, we obtain:

$$\frac{\Delta E_\tau'}{\Delta t'} = \frac{\Delta E}{\Delta t} \frac{1}{v_\tau} v_\tau'$$

or

$$\frac{\Delta E_\tau'}{\Delta t'} \frac{1}{v_\tau'} = \frac{\Delta E}{\Delta t} \frac{1}{v_\tau} \tag{44}$$

We thus see that a Lorentz boost transformation in the instantaneous $\bar{v}_\tau$ direction leaves $\frac{\Delta E_\tau}{\Delta t}\frac{1}{v_\tau}$ invariant. However, a boost transformation $\bar{\omega}$ which is perpendicular to $\bar{v}_\tau$ will not only change $\bar{v}_\tau$ but will change $\frac{\Delta E}{\Delta t}\frac{1}{v_\tau}$. This can be seen by dividing equation (39) by (43) for $\bar{\omega} \perp \bar{v}_\tau$.

$$\frac{\Delta E_\tau'}{\Delta t'} = \frac{(\Delta E_\tau + \bar{p}_\pi \cdot \bar{\omega})\gamma_\omega}{\Delta t\,(1 - \frac{\bar{v}_\tau \cdot \bar{\omega}}{c^2})\gamma_\omega}$$

Now

$$\bar{v}_\tau \cdot \bar{\omega} \equiv 0$$

but

$$\bar{p}_\pi \cdot \bar{\omega} \neq 0 \quad \text{in general.}$$

However when we average over the possible directions of $\bar{p}_\pi$ we have by azimuthal symmetry.

$$\langle \bar{p}_\pi \cdot \bar{\omega} \rangle = 0 \Rightarrow \left\langle \frac{\Delta E_\tau'}{\Delta t'} \right\rangle = \left\langle \frac{\Delta E}{\Delta t} \right\rangle$$

From now on we assume that this average has been performed. Hence for <u>arbitrary</u> boosts: (using $v_\tau' \neq v_\tau$ in the last equation)

$$\left\langle \frac{\Delta E_\tau'}{\Delta t'}\frac{1}{v_\tau'} \right\rangle \neq \left\langle \frac{\Delta E}{\Delta t}\frac{1}{v_\tau} \right\rangle$$

So for simplicity we will henceforth only consider boosts $\bar{u} \parallel v_\tau$, for which:

$$\frac{dE}{dt}\frac{1}{v_\tau} = \text{const.} \tag{45}$$

It is independent of the instantaneous tachyon velocity for these boosts.

This equation is exact for electromagnetic Cherenkov radiation with no restriction on the fact that the direction of $\bar{v}_\tau$ will perform a random walk because of the recoil. It will be of little use because of this random walk. Equation (45) will be valid for each individual segment of the tachyon path. The constant on the right hand side of equation (45) has only been shown to be a constant under Lorentz boosts parallel to the instantaneous tachyon velocity. If the velocities of these segments of path are not parallel to a given direction then there is no reason to conclude that this is the same constant for each segment. In order to conclude that $\frac{dE}{dt}\frac{1}{v_T}$ equals the same constant for a finite portion of the tachyon world line under a group of unidirectional boosts we assume that the direction of the tachyon velocity is approximately a constant.

We see no justification for believing this to be valid for very low tachyon energies. When the tachyon energy is higher and the momentum correspondingly great then we assume that there exists a frame of reference in which the history of the average velocity of a tachyon emitting Cherenkov radiation is unidirectional. Since the magnitude of the tachyon velocity will not be a constant, such a history cannot be supposed to hold in every inertial frame: Lorentz boosting in a direction perpendicular to v will yield a velocity history which is not unidirectional. The existence of a frame in which v is rectilinear (at least for a time) permits us to say that $(dE/dt)\frac{1}{v_T}$ is a constant for boosts parallel to $\hat{v}_T$ This result was also independently found by Jones [72] who also derived it with the limitation that there must exist an inertial frame in which the motion is rectilinear. For the case in which a massive field is being Cherenkov radiated the equation is only valid when the tachyon energy is sufficiently great.

Since an increment of distance along the tachyon path is $dx = v_\tau dt$; dE_τ/dx is invariant under Lorentz boosts. But we require dE/dx to be a function of v_τ only, and the boosts change v_τ, so dE_τ/dx is the same number for all values of v_τ in the history of the tachyon: i.e. it is a constant in time.

$$\frac{dE}{dx} = \text{Const.} \tag{46}$$

From equation (46) and the expression for energy in terms of velocity we have:

$$\frac{d}{dt}\frac{m_\tau c^2}{\sqrt{v^2/c^2-1}} = -fv$$

where f is a constant characterizing the Cherenkov radiation process. Therefore:

$$-\frac{m_\tau v}{(v^2/c^2-1)^{3/2}}\frac{dv}{dt} = -fv$$

Now use the expression for proper distance and integrate:

$$(ds)^2 = (dx)^2 - (c\,dt)^2 = c^2\left(\frac{v^2}{c^2}-1\right)(dt)^2$$

$$\int\frac{dv}{(v^2/c^2-1)} = +\frac{f}{m}\int\left(\frac{v^2}{c^2}-1\right)^{1/2}dt = \frac{fs}{mc}$$

Recognize that $d\coth^{-1}u = -(u^2-1)^{-1}du$ and let $g = -f/mc^2$

Therefore: $\coth^{-1} v/c = -\frac{f}{mc^2}s \equiv gs$

and

$$v = c \coth gs$$

$$dr = \frac{ds}{(v^2/c^2 - 1)^{1/2}} \coth gs = \frac{ds \coth gs}{\left[\left(\frac{\cosh}{\sinh}\right)^2 - 1\right]^{1/2}} = ds \cosh gs$$

Integrate now and choose the origin appropriately:

$$x = \frac{1}{g} \sinh gs$$

x is the distance along the tachyon trajectory, it is a rectilinear distance. Also

$$c\,dt = \frac{ds}{(v^2/c^2 - 1)^{1/2}} = \frac{ds}{\left[\left(\frac{\cosh}{\sinh}\right)^2 - 1\right]^{1/2}} = ds \sinh gs$$

set $t_0 = 0$

Hence

$$ct = -\frac{1}{g} \cosh gs$$

Therefore

$$c^2t^2 - x^2 = g^{-2} \tag{47}$$

We have seen that a properly covariant energy loss rule for a tachyon emitting Cherenkov radiation leads to a kind of hyperbolic motion, but hyperbolic motion with $v > c$. For the usual hyperbolic motion one has (see figure 6a).

$$x^2 - c^2t^2 = g^{-2}$$

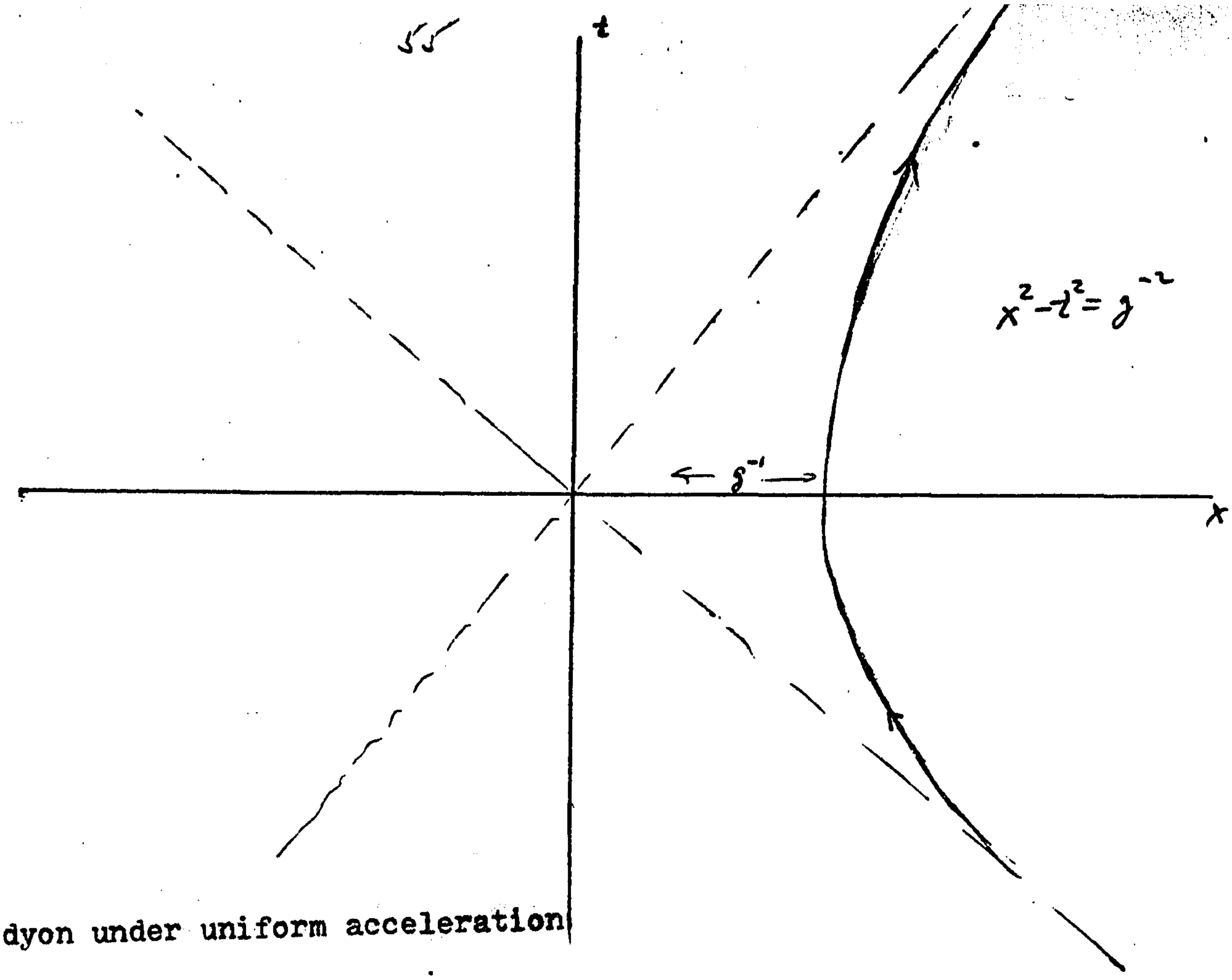
But in (47) effectively the space and time parameters are interchanged. Moreover, equation (47) is based on (46) which, for Cherenkov radiation of a massive field, we have derived only for high energy tachyons. Even for electromagnetic Cherenkov radiation the presence of recoil means that the

Parameter x measured the distance along the tachyon trajectory only when it is linear. Hence, figure 6(b) should be viewed as indicating the progress of a tachyon under uniform acceleration, valid only for large x. Note that the abscissas have different meanings in (6a) and (6b).

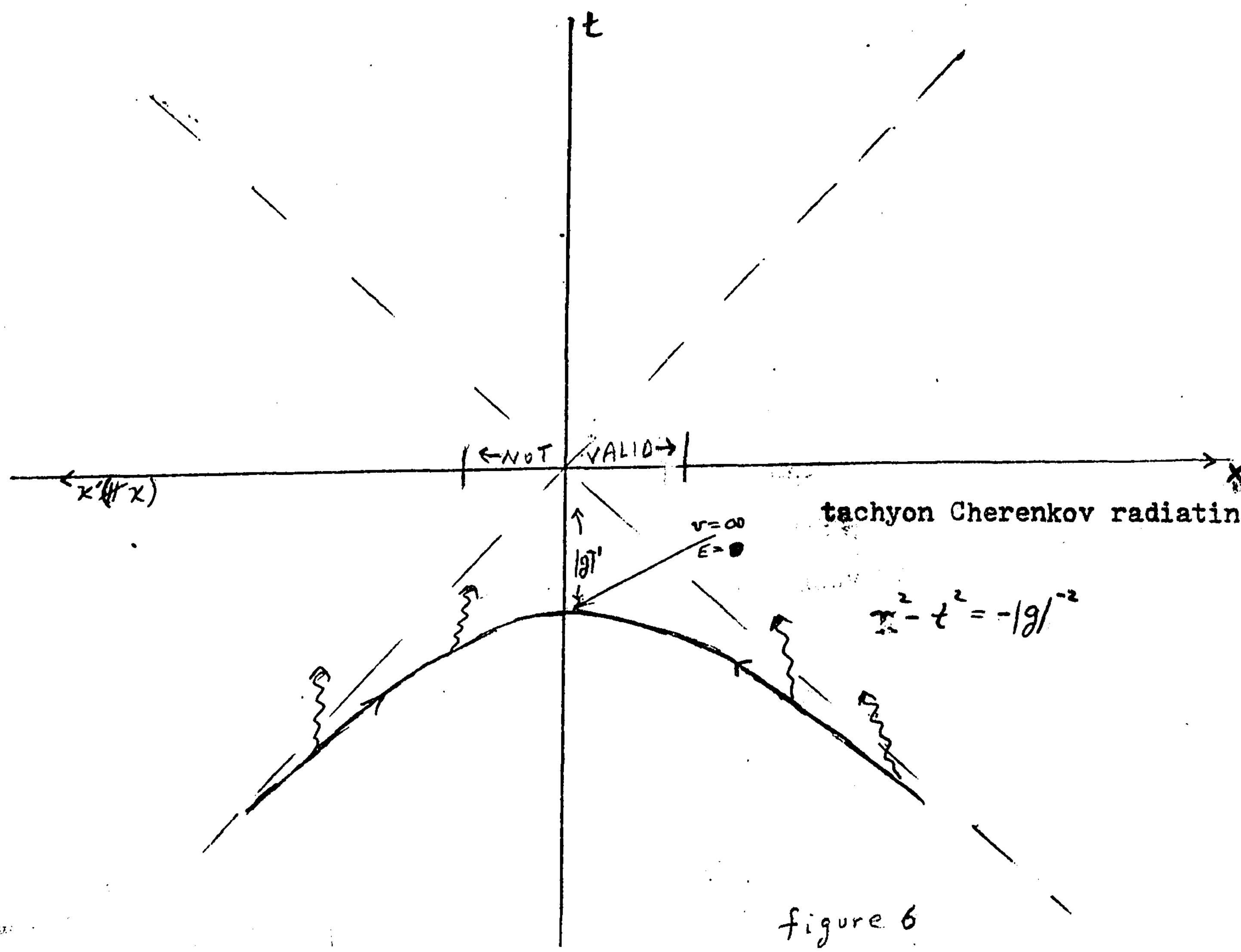
It is apparent that as the tachyon loses energy through radiation its energy falls to zero. It would not be Lorentz covariant to require that the tachyon be absorbed by normal material before E reaches zero, so we must deal with the continuation of the tachyonic motion into the $E<0$ region. The motion in the $E<0$ region can be "reinterpreted" as the motion of an incoming antitachyon which then annihilates the original tachyon. Alternatively, we may choose to allow tachyons to move so that $dt/d\lambda<0$ with λ an affine parameter on the tachyon world line (see, for instance, the equation preceding (47)).

For consistency it seems reasonable to ask that there be a source for the incoming $\bar{\tau}$. There will then always exist another reference frame in which the tachyon would never fall below zero energy and would be destroyed at the second "source".

Figure (7) shows a Minkowski diagram with three inertial frames. In (x,t) the tachyon is emitted at t = 0 and absorbed at a later time t_B after undergoing Cherenkov radiation. In (x',t') a tachyon is created at t = o and another tachyon is created at an earlier time t_B'. The two tachyons annihilate at a time t_c' which is later than both t = 0 and t_B'. In the frame (x'',t'') a tachyon is created at t_B'' and Cherenkov

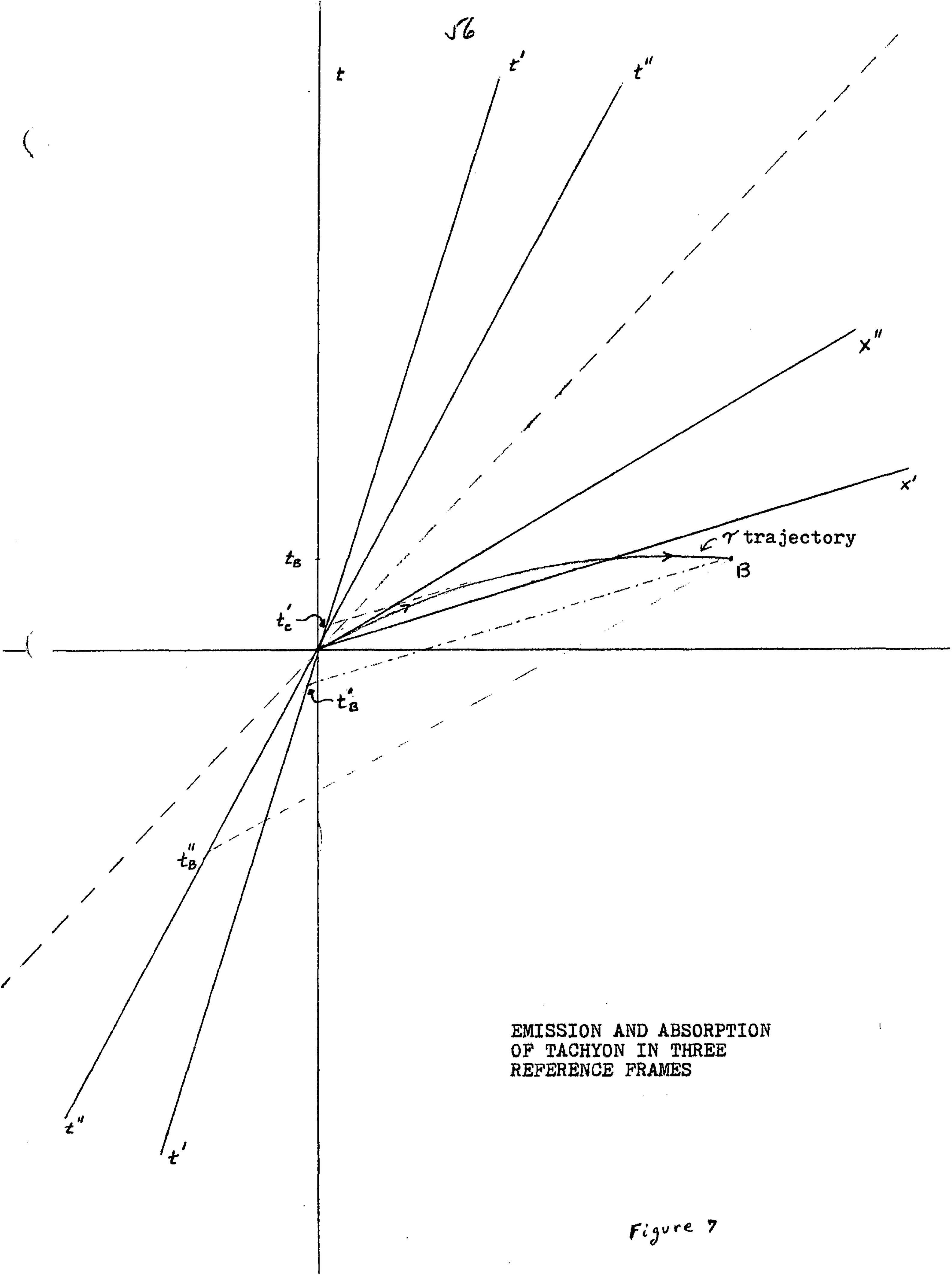


tardyon under uniform acceleration



tachyon Cherenkov radiating

figure 6



EMISSION AND ABSORPTION
OF TACHYON IN THREE
REFERENCE FRAMES

Figure 7

radiates until it is absorbed at t = 0. Thus different observors may give quite different accounts of the same process.

We see that for tachyons which can emit Cherenkov radiation we do not have the freedom to specify arbitrarily the creation of just one tachyon. For consistency we can only specify the complete tachyon path including the events associated with its endpoints, events which are inherently quantum mechanical and not under the control of external agencies.

Because of the difficulties inherent in discussing the trajectory of any quantum mechanical object and particularly that of a tachyon, we give an alternate derivation of the energy loss equation by examining the history of the tachyon in momentum space. We make the same assumptions as before, namely, that the energy loss associated with Cherenkov radiation at any instant depends only on $\bar{p}_r$ and E_r at that instant. We also have to assume that in the observor's frame the motion is rectilinear.

Since there is no preferred reference frame for the tachyon, we expect the energy loss equation to be expressible in a Lorentz covariant way. We expect that there is some way of saying that the "rate" at which "energy" is radiated is a constant, independent of p_r and t.

During a given interval of time the component of a tachyon wave packet with energy and momentum $(E_1, \bar{p}_1)$ radiates an amount of energy ΔE and momentum $\Delta\bar{p}$ and moves to point $(E_2, \bar{p}_2)$ which is constrained to remain on the mass hyperboloid. The invariant distance in four-momentum space is:

$$(\Delta S_E)^2 = (\Delta E)^2 - (c\Delta \bar{p})^2$$

The lapse of invariant proper distance which is associated with this move along the mass hyperboloid is ds. An invariant statement of the constancy of motion along the trajectory in momentum space per unit proper distance is:

$$\frac{dS_E}{ds} = f, \quad \text{with } f \text{ a constant}$$

or

$$\frac{dS_E}{ds} = \frac{\left[\left(\frac{dE}{c\,dp}\right)^2 - 1\right]^{1/2} c\,|d\bar{p}|}{ds}$$

Now use the standard expression for group velocity

$$\frac{dE}{d\bar{p}} = v_g$$

and for proper distance and time use the relation (cf. $ds^2 = dx^2 - c^2dt^2$)

$$ds = c\sqrt{v^2/c^2 - 1}\; dt$$

Therefore

$$\frac{dS_E}{ds} = \frac{\sqrt{v^2/c^2 - 1}\,|d\bar{p}|}{\sqrt{v^2/c^2 - 1}\; dt} = \frac{|d\bar{p}|}{dt} = f$$

We define a phenomenological distance along the tachyon path in terms of v_g and dt, i.e. $dx = v_g\, dt$.

Then we also find:

$$\frac{dS_E}{ds} = \frac{\left[1-\left(d\bar{P}/dE\right)^2\right]^{1/2} dE}{\sqrt{v^2/c^2-1}\; dP/v} = \frac{dE}{dx} = f$$

The tachyon loses a constant amount of energy per unit path length and ~~a constant amount~~ of momentum per unit time; the magnitude of the force of radiation reaction is constant, independent of tachyon speed and invariant under boosts parallel to v_T.

In order to proceed further with the analysis of the effect of Cherenkov emission on tachyon motion we need to evaluate the parameter f. Let us first look at the electromagnetic Cherenkov effect. Then we suppose that f can depend only upon the following dimensional parameters: $ƛ = \frac{\hbar}{m_T c}$, e and c. Then f = $\beta \frac{e^2}{ƛ^2}$, with β a dimensionless parameter, probably of the order of unity.

In a covariant model without arbitrary cutoff factors we would find that β is given by a divergent integral. However we may regard the Cherenkov reaction force as the tachyon analog of the virtual radiative reaction in quantum electrodynamics, a virtual reaction which gives divergent contributions to the electron mass ([70] Cawley). In QED a renormalization procedure is invoked, which renders the electron mass finite but uncalculable. We may similarly replace the divergent f by a finite one, but argue that the numerical value is not calculable, since there will always be sufficiently many free parameters in our renormalized cutoff scheme to allow any value of (f). It is reasonable to suppose, however, that the renormalized β is of the order of unity. Jones [72] has calculated the electromagnetic Cherenkov radiation from a deformable "sphere" and obtained a quite similar result, if the Compton wavelength is interpreted as the radius of the "sphere", in which β is of order unity.

If we use the noncovariant expression of equation (34) of Alvåger and Kreisler in the limit as $v \to c$ or $E_\tau \to \infty$ (where it should be less objectionable because the range of integration is infinite) we find:

$$\lim_{v_\tau \to \infty} \frac{dE}{dx} = -\frac{e^2}{2\chi^2} \tag{49}$$

Our proof of the constancy of dE/dx says that this expression should be valid for all v if it's valid for $v \to c^*$. So we have $\beta = \frac{1}{2}$ in the AK model, based on a point electron with a cutoff E_τ.

To get some idea of the rate of loss associated with Cherenkov radiation we substitute into dE/dx the parameters for an electron of charge Ze. We obtain approximately:

$$\left.\frac{dE}{dx}\right|_{c} = -Z^2 \cdot 10^9 \frac{MeV}{cm.}$$

Hence the range will be of order of magnitude 10^{-9} cm. if the tachyon has an energy of 1MEV. The concept of the Cherenkov range for a tachyon is not well defined or unambiguous. Here, we assume the tachyon is absorbed before it acquires a negative energy with a magnitude far greater than the initial

1 MEV.

We are looking at the tachyon world-line segment in a frame in which both the energy changes associated with ends of the tachyon line are of the order of 1 Mev or less. There are other frames for which this energy condition is violated for the very same tachyon trajectory; in such frames the range may be much greater than the "symmetrical" range defined above.

Chapter IV

The Field Emitted During a Generalized Cherenkov Process

In 1937 P. A. Cherenkov extensively studied the radiation now known by his name. (see [58] Jelley for a complete list of references) This radiation is emitted by an electrically charged particle travelling with a velocity which exceeds the phase velocity of light in the medium. It can be viewed as a cooperative phenomenon which results from the acceleration of, and consequent radiation by the atomic electrons in the medium, [62] Jackson.

This type of radiation was previously investigated in pre-relativity days by [89] Thomson, [92] Heaviside and [04] Sommerfeld. Not knowing that he was not supposed to consider particles exceeding the velocity of light in a vacuum, in 1904 Sommerfeld calculated the properties of the radiation from various charge configurations moving with a constant v greater than c.

Perhaps the simplest way in which Cherenkov radiation is understood is as a shock wave like the sonic boom of a jet. For v greater than c the spherically propagating field interferes with itself constructively, forming a radiation field travelling at an angle with respect to v defined by $\cos \theta = c/v$ (See figure (8))

To calculate the Cherenkov angle we may determine the two retarded positions of the charge contributing fields which add constructively on the Cherenkov cone. Since ordinary Cherenkov radiation may be interpreted as a collective phenomenon of the medium and no such interpretation is possible for a vacuum, some investigators reject the possibility of a Cherenkov radiation in a vacuum, given that there might exist charged tachyons.

Because of the simplicity of the geometric construction which helps one to understand the formation of the shock front for electromagnetic Cherenkov radiation, others fail to consider the possibility of Cherenkov radiation of fields having massive quanta.

In the following we investigate the properties of generalized Cherenkov radiation of a field with massive quanta which we call "pi" particles.

We wish to gain a generally valid understanding of the phenomenon as far as possible unrestricted by any assumptions associated with a particular model. Later we will particularize to one model for definiteness. We first consider how it is possible for a generalized charge, static in its "rest frame", undergoing uniform rectilinear motion, to generate excitations in the field to which it couples. For the case of the massless field the formation of the shock wave follows the geometric construction alluded to above. For a massive field the situation is more confusing. Seemingly it is different in two ways. First of all, the short range of the field makes us question the geometric approach. And generally the fact that the pi mass is not zero means that the dispersion relation ω vs. k for pi waves is nonlinear, giving a variable phase velocity and a variable group velocity for the pi-waves. It is not clear how pi-waves of different k will constructively interfere to produce a retarded field attached to the superluminal source.

Compare the retarded Green's functions for a massless field where the retardation is easy to see:

$$\Delta^{ret} = \frac{\delta(x^2)}{2\pi}\Theta(x^0) = \frac{\delta(|\bar{x}|-ct)}{4\pi|\bar{x}|}$$

with the corresponding massive Green's function (for x^2 smaller than the Compton wavelength)

$$\Delta^{ret} \simeq \left[\frac{\delta(x^2)}{2\pi} + \frac{\Theta(x^2)}{2\pi}\left(\frac{m^2}{4} + \frac{x^2 m^4}{32} + \cdots\right)\right]\Theta(x^0)$$

The $\Theta(x^2)$ function says that no signals exceed the speed of light. However, all $v \leq c$ are apparently represented.

Leaving the geometric constructive interference approach, which is not easily visualized in the massive field case, we seek further enlightenment in the Fourier transform picture, i.e. in momentum space.

In order to see what modes of the massive field a particle with constant velocity can couple to, we consider the simplest situation. A point particle with charge g is adiabatically switched on and then off.

$$\rho(\bar{r},t) = g\,\delta^3(\bar{r}-\bar{v}t)\,e^{-\alpha|t|} \tag{1}$$

Taking the Fourier transform:

$$\rho(\bar{k},\omega) = \int_{-\infty}^{\infty} dt\, d^3r\, \rho(\bar{r},t)\,e^{-i(\bar{k}\cdot\bar{r}-\omega t)} \tag{2}$$

We obtain:

$$\rho(k,\omega) = g\int_{-\infty}^{\infty} dt\, e^{-\alpha|t| + i\omega t}\, e^{-i\bar{k}\cdot\bar{v}t}$$

$$= g\left[\frac{1}{\alpha + i(\omega - \bar{k}\cdot\bar{v})} + \frac{1}{\alpha - i(\omega - \bar{k}\cdot\bar{v})}\right]$$

$$= g\left[\frac{2\alpha}{\alpha^2 + (\omega - \bar{k}\cdot\bar{v})^2}\right]$$

Hence, letting α go to zero:

$$\rho(k,\omega) = 2\pi g\, \delta(\omega - \bar{k}\cdot\bar{v}) \tag{3}$$

For the Fourier components of ρ which couple to real quanta of the "pi" field ω and k must satisfy:

$$\hbar^2\omega_\pi^2 = \hbar^2 k c^2 + m_\pi^2 c^4 \tag{4}$$

We set $\hbar = c = 1$, although sometimes the factors of c are reinstated where deemed helpful.

Hence we might have massive Cherenkov radiation when the nonzero Fourier components of the source, satisfying the relation

$$\omega = \bar{k}\cdot\bar{v}_r \tag{5}$$

match the modes of the field (equation 4).

Combining these two equations we find:

$$m_\pi^2 c^4 + c^2 k^2 = (\bar{k}\cdot\bar{v})^2 \tag{6}$$

$$= (k_\pi v_r \cos\theta)^2$$

Solving for k:

$$|\bar{k}| = \frac{m_\pi c}{\sqrt{\frac{v_\tau^2 \cos^2\theta}{c^2} - 1}} \tag{7}$$

It is seen that the denominator must be imaginary unless the velocity of the source, v_T; is greater than c, in particular:

$$\frac{v_\tau^2 \cos^2\theta}{c^2} > 1 \tag{8}$$

Looking again at the delta function above, where ω , k satisfy equation (5) and using the well known relation for a free particle

$$v_{group} = \frac{\bar{p}\, c^2}{E} = \frac{k_\pi}{\omega_\pi} c^2$$

we find that:

$$\frac{\bar{v}_\pi \cdot \bar{v}_\tau}{c^2} = 1 \tag{9}$$

We solve for cos θ, the cosine of the angle between the velocity of the source and the direction of the emitted generalized Cherenkov radiation:

$$\cos\theta = \frac{c^2}{v_\pi v_\tau} \tag{10}$$

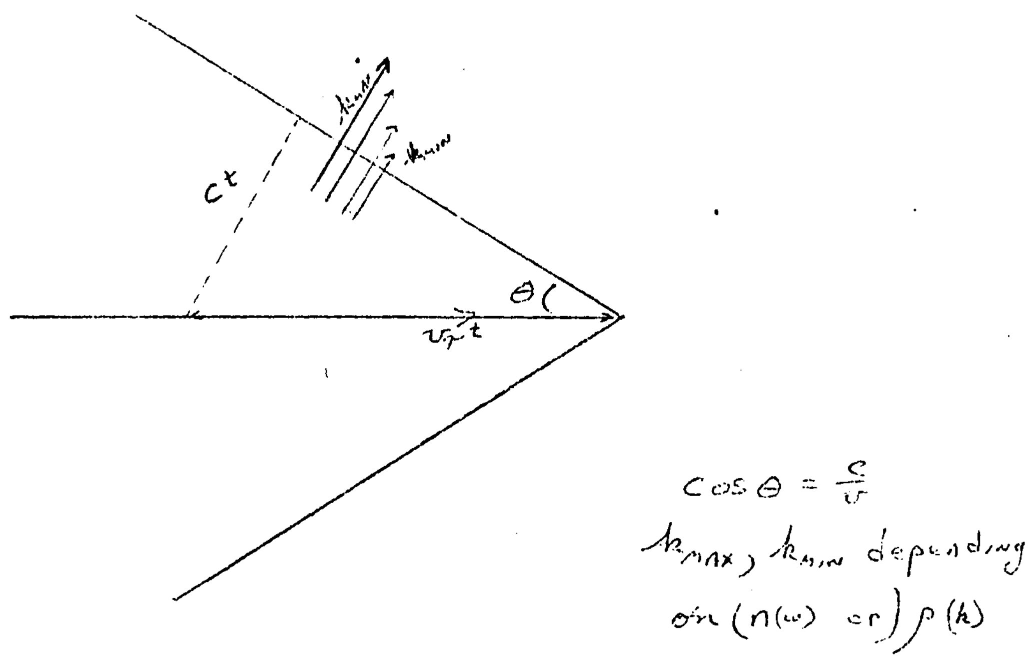
This reduces to the usual result for electromagnetic Cherenkov radiation when we take v_π=c (and of course m=0)

$$\cos\theta = c/v_\tau$$

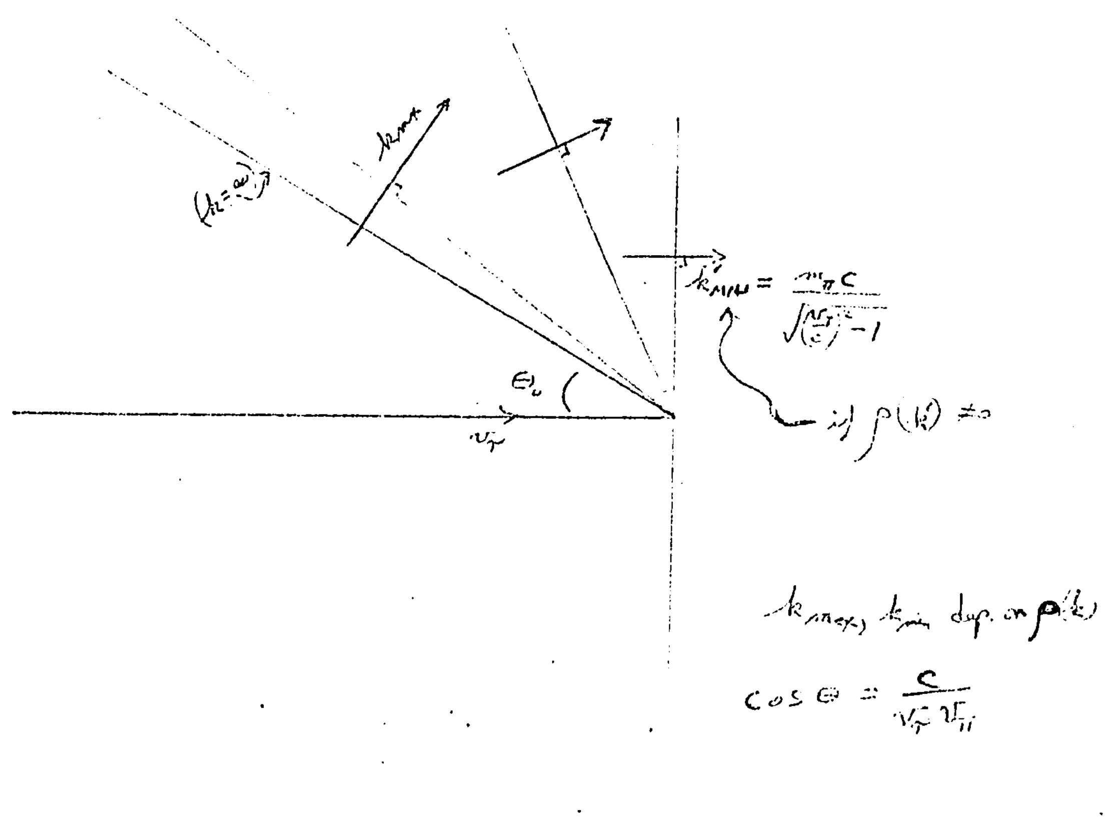
Note that for electromagnetic Cherenkov radiation there is only one angle which satisfies the conditions because the photon can have only one velocity in equation (9) or equation (10) .

However, for the massive field v_π can take on a range of

CHARACTERISTICS OF GENERALIZED Č RAD.



ORDINARY E-M ČERENKOV RADIATION (Here shown in VAC.)



CHERENKOV RADIATION OF MASSIVE FIELD (RECOILLESS)

figure 8

values, and so also can θ. (figure (8)) From equation (8) we see that k is real for θ lying between 0 and $\cos^{-1}\frac{c}{v_\tau}$, the latter being the usual electromagnetic Cherenkov angle. This corresponds to allowing the velocity of emitted π's to range from c^2/v_τ to c. For a particle satisfying the Klein-Gordon equation $v_{phase} \cdot v_{group} = c^2$.

Hence, the π emitted in the forward direction by the tachyon source has group velocity equal to the tachyon phase velocity and the minimum $|\bar{k}|$ satisfying equation (7)

$$k_{min} = \frac{m_\pi c}{\sqrt{\frac{v_\tau^2}{c^2} - 1}}$$

Combining equations (7) and (10) gives the usual result for the momentum of the π. We also easily see that

$$k_{min} = \frac{m_\pi c}{\sqrt{\frac{v_\tau^2}{c^2} - 1}} \xrightarrow[v_\tau \to \infty]{} 0$$

$$\xrightarrow[v_\tau \to c^+]{} \infty$$

and

$$k_{max} = \infty$$

A number of more general remarks can be made concerning the relations which ω and k must satisfy. Writing $\bar{v}_\pi$ and $\bar{v}_\tau$ in terms of momentum and energy, equation (9) becomes:

$$1 = \frac{\bar{p}_\pi}{E_\pi} \cdot \frac{\bar{p}_\tau}{E_\tau} c^2$$

or more succinctly:

$$p_\mu^\pi p_\mu^\tau = 0 \qquad (11)$$

In other words, the four-momenta are orthogonal. Now we can see from another viewpoint that p_τ^μ has to be spacelike for $\check{C}$ radiation to be possible. If p_π^μ is timelike no other real timelike or null four-vector can be orthogonal to it. In a medium however, the insertion of factors of $n(\omega)$ will imply that p_γ^μ will be spacelike where n is the index of refraction. For photons

$$p_\gamma^\mu = \hbar\left(\omega, \frac{n\omega}{c}\right)$$

$$p_\mu^\gamma p_\mu^\gamma = \omega^2(1-n^2) < 0 \quad \text{if} \quad n > 1$$

Further, since p_τ^μ <u>is</u> spacelike, we note also that there are spacelike four-vectors which are orthogonal to it. This raises the possibility that a tachyon field might emit Cherenkov radiation into other tachyon modes. We shall not explore that possibility in this thesis.

Now note that equation (9) is the condition for a Lorentz transformation with boost velocity $\overline{v_\pi}$ to an inertial frame in which the source has infinite velocity and zero energy. This frame is also the rest frame of the emitted π and the frame in which the source would be a pure current. (see Appendix).

Note that for a tachyon

$$p_\mu^\tau p_\mu^\tau = -|m_\tau|^2 \equiv -m_\tau^2 \tag{12}$$

Hence:

$$p_\mu^\tau \, d p_\mu^\tau = -m_\tau dm_\tau \tag{13}$$

Now if the tachyon does not change its internal state

$$dm = 0 \tag{14}$$

By conservation of momentum, the four momentum of the emitted π must equal the negative of the change in p_μ^τ. Hence:

$$d p_\mu^\tau = - p_\mu^\pi \tag{15}$$

Therefore combining (13), (14), and (15):

$$p_\mu^\tau p_\mu^\pi = 0 \quad , \tag{16}$$

and we obtain equation (11) again. In this argument we assumed that $p_\mu^\pi \ll p_\mu^\tau$, i.e. we ignored tachyon recoil.

The Lorentz transform of the energy of the π is

$$E' = \gamma \left[E - \bar{p}_\pi \cdot \bar{u} \right]$$

So if we, for the moment, wanted to consider the possibility of superluminal Lorentz transformations, we see that if $u = v_\tau$ then by equation (9) or (11) we have:

$$E' = 0$$

Hence the energy of the pi particle which can be Cherenkov radiated is zero in the rest frame of the source, as needed by conservation of energy since the source doesn't change its intrinsic properties.

Equation (10) differs from the equation for cos θ derived in Chapter III. The difference can be explained by our present use of equation (1) an (13) and (14) which tacitly assume no recoil or m_τ and $\bar{p}_\tau$ equal to infinity.

To conclude this section we note that conditions on f_μ^τ which have been derived may be necessary but certainly are not sufficient for the existence of massive Cherenkov radiation. The full justification of these results must rest on a detailed calculation of the field generated by a source. However, some insight has been gained into the characteristics of the radiation to be expected from a source whose velocity exceeds that of light. In particular, the relation of angle of emission to momentum of the π was found. We also demonstrated the existence of a minimum for the momentum of a pi particle which can be emitted as Cherenkov radiation if $m_\pi \neq 0$.

We have previously investigated the possibility of a Cherenkov radiation of massive particles from the viewpoint of particle kinematics and from the viewpoint of wave field resonance. Having demonstrated the kinematic possibility of such generalized Cherenkov radiation and found the relation between angle of emission and the energy-momentum four vector of the emitted particles, we now need to look at the dynamics of particular models for the coupling.

One can identify a number of candidates for a detailed model calculation of generalized Cherenkov radiation.

Model A: Stochastic Spinless Quantum Theory

One can calculate the transition probability, figure (9), $\tau \rightarrow \tau + \pi$, and differentiate with respect to time to get the rate of emission of energy. Since the tachyon cannot make a transition into a state which cannot Cherenkov radiate, the energy emission is a continuous multi-particle process yielding a characteristic dependence on time. In such a calculation, the δ function which expresses conservation of energy and momentum determines the Cherenkov angle as in Chapter III.

This calculation can take recoil into account. The results yield the rate of loss of the tachyon energy ~~of the emitted radiation energy~~, and the polarization and frequency distribution of the emitted radiation.

For an electron in a medium there are three possibilities for a maximum frequency cutoff. (i) One can certainly use the electron energy as the cutoff since the particle cannot lose more than its total energy. (ii) The index of refraction is a function of ω and in general n approaches unity when ω becomes large enough. There will be an ω_{max} determined by $n(\omega)$ beyond which $v_e < c/n(\omega)$. There will be no cherenkov radiation for those frequencies exceeding ω_{max} since $\cos\theta = \frac{c}{n v_e}$ will then exceed one. (iii) There is also a maximum frequency of Cherenkov radiation in a medium determined by the conservation laws. Solving equation III - (33) for ω:

$$\omega = \frac{2|\bar{p}_e|c}{n\hbar} \frac{\left(\cos\theta - \frac{c}{vn}\right)}{1 - 1/n^2}$$

We see that for a given electron velocity and index of refraction, ω reaches a maximum when $\cos\theta=1$ ([57] Sokolov):

$$\omega_{max} = \frac{2|\bar{p}|}{n\hbar} \frac{1 - \frac{c}{vn}}{1 - 1/n^2} \tag{17}$$

For a tachyon in the vacuum none of these frequency cutoffs can be used. The noncovariance of (i) has been discussed in Chapter III. Neither (ii) nor (iii) are relevant since they both go to infinity when $v > c$ and $n=1$.

Two and three are also no good for a prearranged disturbance which exceeds the velocity of light in vacuum such as discussed by Bolotovskii and Ginzburg [72]. Hence, a form factor for the tachyon must be introduced. We shall perform such a calculation, taking special cognizance of the form factor problem.

Model B: A Prescribed Source

In the event that the field is coupled linearly to a prescribed current or source, with $H_{int} = \int \rho \phi_{\pi}$, one obtains a linear equation of motion for ϕ_{π}. The solution for the field can be written down in terms of standard Green's functions, even in the case of massive Cherenkov radiation. This method is similar to the classical solution for electromagnetic Cherenkov radiation. It has the added virtue of providing, in addition to ϕ^{out}, the interpolating field including the virtual π's or γ's. Therefore it yields the field behind the tachyon and hence, the force exerted on a stationary "charge". The rate of energy emitted by the source may for instance be calculated from integrating the stress tensor $T^{\mu\nu}$ for the emitted field. The solution for ϕ_{π} makes evident some interesting features of massive Cherenkov radiation; for example, the field ϕ_{π} is in a coherent quantum state. We shall carry out such a calculation also.

Model C: Spin Effects

The effect of the spin of the source on the emitted field for ordinary Cherenkov radiation has been calculated for various spins up to two, ([58] Jelley). All unitary irreducible representations of the inhomogeneous Lorentz group for spacelike four-momentum, except the scalar case, are infinite dimensional in the spin variable, ([58] Shirokov). This might cmmpletely change the character of the Cherenkov radiation, but we shall not pursue such models.

Model D: NonYukawa Coupling, etc.

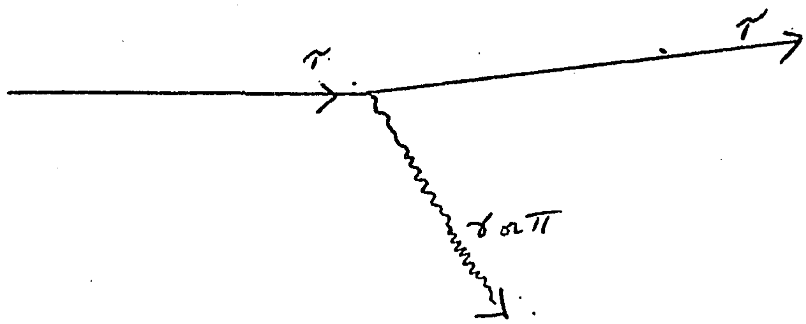
Higher order contributions to the Cherenkov radiation might be calculated. For instance, for a Boson current one can include the simultaneous emission of two photons or possibly massive quanta ("seagulls") figure (9b). Also contributions from virtual pairs could be included to this order. Figure (9c). We shall not pursue such models, either.

Model E: An $\mathcal{S}$-Matrix Model

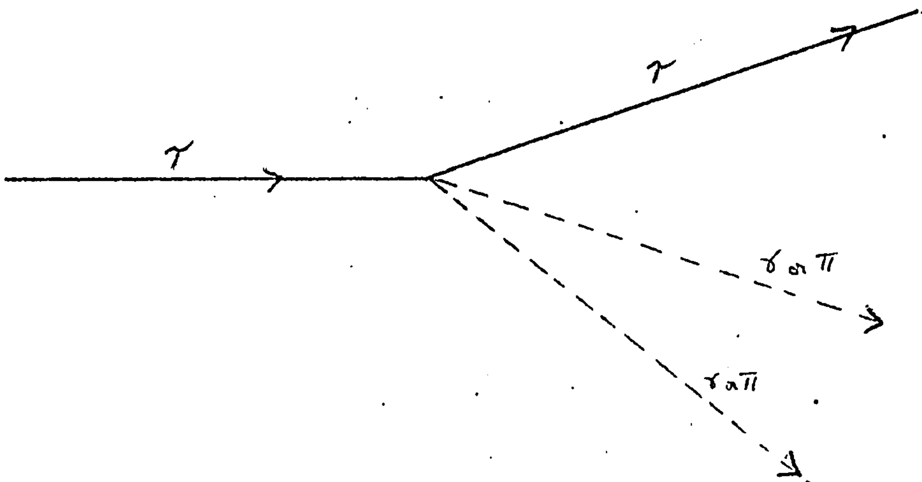
Such a model would allow us to avoid the stochastic assumptions of model A, and would allow us to investigate the degree to which a proper quantum treatment of the tachyonic degrees of freedom would alter the coherence properties of the emitted Cherenkov radiation. On the one hand there is the coherent aspect of the emission resulting from model B. This is to be compared with coherent radio waves resulting from macroscopic currents of electrons. Or it is to be compared to the emission from a laser where stimulated emission and feedback produced by the mirrors give rise to what can be visual-

POSSIBLE PERTURBATION CALCULATIONS

(1) (a)

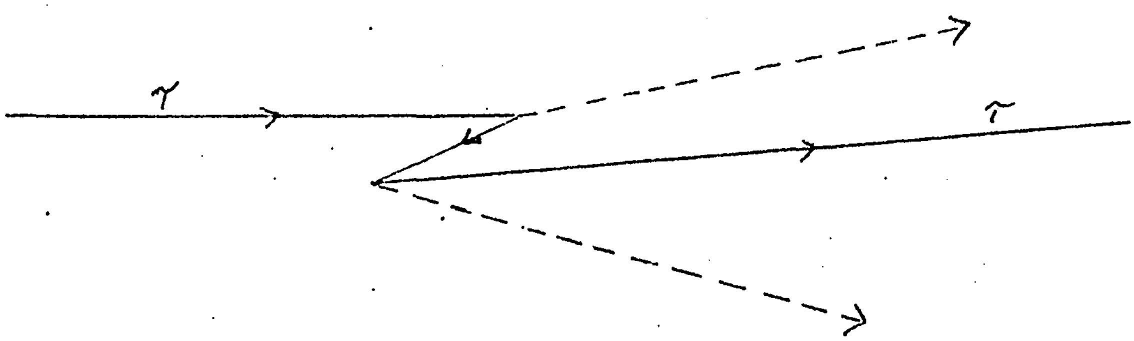


(2) (b)



SPIN 0 BOSON (TACHYON OR PARTICLE IN MEDIUM)

(3) (c)



VIRTUAL PAIRS figure 9

ized as a net atomic current of macroscopic size associated with a large number of atoms.

On the other hand, there is the postulated incoherence of the radiation emitted in model A, an incoherence we associate with the recoil-induced alterations in the state of motion of the tachyonic source.

With Cherenkov radiation, one encounters a situation that may be of an intermediate sort. Because of the great speed of the Cherenkov radiating particle one might expect some aspects of a net classical current with quantum fluctuations superimposed. These classical features might be similar to those of synchrotron or bremsstrahlung radiation, for example.

We now calculate the generalized Cherenkov radiation of a massive field using Model B. We assume that the scalar field, which we call "π", is coupled to a superluminal prescribed source, which we represent by a c number, that is, by something that commutes with all operators of the theory, and which has a nonvanishing expectation value in the states of interest. As is well known, the field generated by a prescribed c number source is in a "classical" or "coherent" state. [68] Klauder and Sudarshan, [62] Henley and Thirring (hereafter H+T) Hence, the problem is the essentially classical one of finding the field ϕ_π which is generated linearly by a given ρ . The Green's function (H. and T.) with the boundary condition appropriate to outgoing waves will be employed. After finding ϕ^{out} we will determine the creation operator for emitted

π particles and then, $\frac{dn_k}{dt}$, the number emitted per unit time at various momenta. This model does not include recoil; we will show that the kinematic analysis given earlier in this Chapter is appropriate for determining the coupling to the modes of the field. The assumptions are appropriate for a model of a very massive, classical tachyon with a given charge distribution.

Further analysis, after obtaining $\frac{dn_k}{dt}$, will be applicable to model A also.

The Hamiltonian for the pi field, including the interaction with the source, is: (H and T)

$$H = H_0 + H_1 = \frac{1}{2}\int d^3r \left[\dot{\phi}^2 + (\nabla\phi)^2 + m_\pi^2\phi^2 - 2g\rho(\bar{r},t)\phi\right] \tag{18}$$

We consider the source to be undergoing uniform superluminal translation with a velocity $\bar{v}$ in the $\hat{z}$ direction. Since the source (and field) are considered to be scalar, we have ([61] Schweber):

$$\rho'(x') = \rho(x) \quad \text{where } x'^\mu = \Lambda^\mu{}_\nu x^\nu \tag{19}$$

If the source velocity were less than c we could use this equation to find the expression for the moving source in terms of a static charge distribution in the rest frame of the source:

$$\rho_v(x,y,z,t) = \rho_0(\Lambda^{-1}\bar{x}) \tag{20}$$

where Λ^{-1} denotes a Lorentz transformation to the source rest frame, and ρ_0 is the rest frame charge distribution, assumed independent of time. However, for a superluminal source we do not have the freedom to make a Lorentz boost transformation to a nonexistent superluminal inertial frame. Instead, we take as our standard reference frame one in which the tachyon energy is zero.

In addition, form factors are generally taken to be functions of $t \equiv (p' - p)^2$ which in this case would equal m_T^2. Hence, the form factor, roughly corresponding to the Fourier transform of the charge distribution, would give no cutoff to the Cherenkov radiation.

The resolution of these difficulties is found in appendix (C), where we justify writing (note $\gamma \rightarrow |\gamma| \equiv \frac{1}{\sqrt{v^2-1}}$):

$$\rho_v'(x, y, z, t) = \rho_0\left(x, y, (z - vt)|\gamma_v|\right) \tag{21}$$

The "0" in ρ_0 refers to the fact that $E_T = 0$; i.e. $v_T = \infty$ in the standard frame.

We find that certain ambiguities arise if we attempt to switch the source on adiabatically, because of the existence of the Cherenkov radiation which we are trying to calculate. The source will be assumed to be switched on suddenly at $t = -T/2$ and switched off suddenly at $t = +T/2$. At first we do not assume that the charge exists at the other times. In appendix (D) we take this into account and calculate the creation (inner bremsstrahlung) radiation which results.

To calculate the outgoing field, we use the equation (H & T)

$$\phi^{out}(\bar{r}, t) = \phi^{in}(\bar{r}, t) + \iint_{-\infty}^{\infty} \Delta(\bar{r} - \bar{r}', t - t')\rho(\bar{r}', t')\, d^3r'\, dt' \tag{22}$$

where the Green's function is:

$$\Delta = \Delta^{retarded} - \Delta^{advanced} = \frac{1}{(2\pi)^3}\int d^3k \frac{e^{i k\cdot(\bar r - \bar r')}}{\omega_k} \sin\omega_k(t-t') \tag{23}$$

and

$$\rho(\bar r, t) = \rho_0\left(x, y, (z-vt)|\gamma|\right)\left[\Theta(t+T/2) - \Theta(t-T/2)\right] \tag{24}$$

Since ϕ^{in} is assumed to be zero, except for the zero point fluctuation of the field, we will drop it. ϕ^{in} is still needed, of course, to maintain the canonical commutation relations of the field. Inserting (23) and (24) in (22) we obtain:

$$\phi^{out}(\bar r, t) = \frac{1}{(2\pi)^3}\int d^3\bar r' \int_{-T/2}^{T/2} dt' \frac{d^3k\, e^{i\bar k\cdot(\bar r - \bar r')}}{\omega_k} \sin\omega_k(t-t')\, \rho_0\left(x', y', (z'-vt')|\gamma|\right) \tag{25}$$

In the integral over $\bar r'$ use the dummy variable for z'.

$$\zeta \equiv (z'-vt')|\gamma| \quad \text{or} \quad z' = \frac{\zeta}{|\gamma|} + vt' \text{ and } dz' = \frac{d\zeta}{|\gamma|}$$

We obtain:

$$\phi^{out}(\bar r, t) = \frac{\sqrt{\frac{v^2}{c^2}-1}}{(2\pi)^3}\int_{-T/2}^{T/2} dt'\, d^3k \int \frac{dx'\,dy'\,d\zeta}{\omega_k} e^{i\bar k\cdot\bar r}\, e^{-i\left(k_x x' + k_y y' + \frac{k_z \zeta}{|\gamma|}\right) - i k_z v t'} \rho_0(x', y', \zeta) \sin\omega_k(t-t')$$

Hence, using the Fourier transform of ρ_0,

$$\phi^{out}(\bar r, t) = \frac{\sqrt{\frac{v^2}{c^2}-1}}{(2\pi)^3}\int_{-T/2}^{T/2} dt' \int \frac{d^3k}{\omega_k} e^{i\bar k\cdot\bar r}\, \frac{e^{i\omega_k(t-t')} - e^{-i\omega_k(t-t')}}{2i}\, \rho_0\left(\bar k_\perp, \frac{k_z}{|\gamma|}\right) e^{-i k_z v t'}$$

Now performing the integral over time:

$$\phi^{out}(\bar{r},t) = \frac{\sqrt{\frac{v^2}{c^2}-1}}{(2\pi)^3}\int \frac{d^3k}{\omega_k} e^{i\bar{k}\cdot\bar{r}} \left[e^{i\omega_k t}\frac{\sin(\omega_k+k_z v)T/2}{i(\omega_k+k_z v)} - e^{-i\omega_k t}\frac{\sin(\omega_k-k_z v)T/2}{i(\omega_k-k_z v)}\right] \rho_0\left(\bar{k}_\perp, \frac{k_z}{|\gamma|}\right) \tag{26}$$

We see that this has the correct time dependence for free radiation of the field ϕ as expected from the standard derivation of Δ, with the contour of integration in k^0, insuring the correct boundary conditions on Δ^{ret} and Δ^{adv}, respectively.

We can obtain a_k by using the relation: [65] Barton:

$$a_k^{out} = i\frac{1}{(2\pi)^{3/2}}\int d^3\bar{r}\,\frac{e^{-ikx}}{(2\omega_k)^{1/2}}\overleftrightarrow{\partial_0}\,\phi^{out}(\bar{r},t)$$

or (H and T) from inspecting the form of ϕ^{out}

$$\phi = \frac{1}{(2\pi)^{3/2}}\int d^3k\,\frac{e^{i\bar{k}\cdot\bar{r}-\omega_k t}a_k + e^{-i\bar{k}\cdot\bar{r}+\omega_k t}a_k^+}{\sqrt{2\omega_k}} \tag{27}$$

With a change of variable in the coefficient of $e^{i\omega_k t}$ so that $\bar{k}\to-\bar{k}$ (26) reaches the form of (27) and then we obtain:

$$a_k^{out} = \frac{i\sqrt{2}\sqrt{\frac{v_T^2}{c^2}-1}}{(2\pi)^{3/2}\sqrt{\omega_k}}\rho_0\left(\bar{k}_\perp, \frac{k_z}{|\gamma|}\right)\frac{\sin(\omega_k-k_z v)T/2}{(\omega_k-k_z v)} \tag{28}$$

Hence from (28)

$$a_k^{+out}a_k^{out} = \frac{2\left(\frac{v_T^2}{c^2}-1\right)}{(2\pi)^3\omega_k}\left|\rho_0\left(\bar{k}_\perp, \frac{k_z}{|\gamma|}\right)\right|^2\left[\frac{\sin(\omega_k-k_z v)\frac{T}{2}}{(\omega_k-k_z v)}\right]^2 \tag{29}$$

Now for large T, ([61] Messiah)

$$\left[\frac{\sin\left(\omega_k - k_z v\right)T/2}{\left(\omega_k - k_z v\right)}\right]^2 \approx \frac{\pi}{2}\, T\, \delta\left(\omega_k - k_z v\right) \tag{30}$$

Hence:

$$\frac{dn_k}{dt} = \frac{d}{dt}\left(a_k^+ a_k\right) = \frac{\left(\frac{v^2}{c^2} - 1\right)}{8\pi^2 \omega_k} \left|\rho_0\left(\bar{k}_\perp, \frac{k_z}{|\gamma|}\right)\right|^2 \delta\left(\omega_k - k_z v\right) \tag{31}$$

The δ function insures massive Cherenkov radiation at the appropriate angle for each value of ω_k. Multiplying by ω_k we obtain the rate of Cherenkov radiation energy loss at each angle.

We note that, as expected, the delta function is the familiar kinematic one. Hence, the kinematic analysis we performed previously gives the relation between the angle of emission and the energy of the emitted particles, and it predicts the existence of k_{min}.

In order to calculate the total energy radiated per unit time, multiply by ω_k and integrate over all wave vectors:

$$\frac{dE}{dt} = \frac{\left(\frac{v^2}{c^2} - 1\right)}{8\pi^2} \int d^3k \left|\rho_0\left(\bar{k}_\perp, \frac{k_z}{|\gamma|}\right)\right|^2 \delta\left(\omega_k - k_z v\right) \tag{32}$$

Assume that the charge distribution of the source is cylindrically symmetric (as in appendix (C)) and use the standard relation ([61] Messiah).

$$\delta(g(x)) = \sum_n \frac{\delta(x-x_n)}{|g'(x_n)|} \quad \text{where} \quad \begin{matrix} g(x_n)=0 \\ g'(x_n)\neq 0 \end{matrix}$$

in order to perform the integration over $k_\perp \equiv \sqrt{k_x^2+k_y^2}$. We have for the integral. (ϕ = AZIMUTHAL ANGLE)

$$\int |\rho_0|^2 \frac{\delta\left(k_\perp - \sqrt{(v^2-1)k_z^2 - m_\pi^2}\right)}{|k_\perp/\omega_k|} k_\perp \, dk_\perp \, d\phi \tag{33}$$

since we used

$$\frac{d\omega_k}{dk_\perp} = \frac{k_\perp}{\omega_k}$$

and

$$\delta(\omega_k - k_z v) \Longrightarrow \omega_k^2 = k_\perp^2 + k_z^2 + m_\pi^2 = k_z^2 v^2$$
$$\Longrightarrow k_\perp = \sqrt{(v^2-1)k_z^2 - m_\pi^2}$$

The δ function also implies $\omega_k = k_z v$ in (33) and $\int d\phi = 2\pi$. Using these in (33), we obtain for (32)

$$\frac{dE}{dt} = \frac{\left(v^2/c^2 - 1\right) v}{4\pi} \int dk_z \, k_z \left|\rho_0\left(\sqrt{(v^2-1)k_z^2 - m_\pi^2}, \frac{k_z}{|\gamma|}\right)\right|^2 \tag{34}$$

At this point, we make some observations on equation (34). From the argument of ρ we see that $k_\perp$ goes to zero when:

$$k_{min} = k_z = \frac{m_\pi}{\sqrt{\frac{v_T^2}{c^2} - 1}} \tag{35}$$

Note that $k_{min} \to 0$ if $m_\pi \to 0$, e.g. for electromagnetic or gravitational Cherenkov radiation. Also k-min $\longrightarrow$ 0 when $v_r \to \infty$.

When v_r increases we also see from Equation (34) that k_{max} as determined by ρ also decreases. Of course there need not be a sharp cutoff, but as v increases the high frequency coupling is diminished because of the Lorentz <u>expansion</u> of the source.

Now make a change of variable in equation (34)

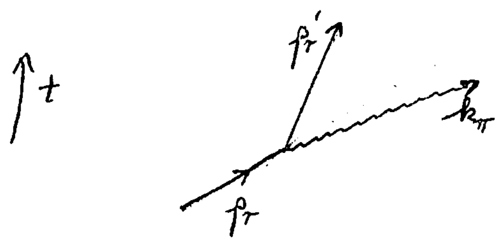
$$k_z' = \frac{k_z}{|\gamma|} \Rightarrow k_{min}' = m_\pi$$

And we obtain:

$$\boxed{\frac{dE}{dt} = \frac{v}{4\pi} \int_{m_\pi}^{\infty} k_z' \, dk_z' \left| \rho_0 \left(\sqrt{k_z'^2 - m_\pi^2}, k_z' \right) \right|^2} \tag{36}$$

All velocity dependence has been removed from the integral. We thus obtain the velocity dependence predicted in Chapter III on the basis of Lorentz covariance.

Now look at the radiated field using model A. We assume that the tachyon changes its external state of motion (it recoils) but that it does not change its "internal" state of excitation during the emission of a single "π" particle. The process we calculate is diagramatically represented thus:



The Hamiltonian is:

$$H = H_\tau + H_\pi + H_{int}$$

where

$$H_\tau = \frac{1}{2}\int d^3\bar{r}\left[\dot{\psi}^2 + (\nabla\psi)^2 - m_\tau^2\,\psi^2\right]$$

$$H_\pi = \frac{1}{2}\int d^3\bar{r}\left[\dot{\phi}^2 + (\nabla\phi)^2 + m_\pi^2\,\phi^2\right]$$

$$H_{int} = -\int d^3\bar{r}\;\;\eta(\bar{r},t)\,\phi(\bar{r},t)$$

and

$$\eta(\bar{r},t) = \int \psi^\dagger(y)\,\psi(y)\,K(x-y)\,d^4y$$

We assume the usual boson commutation relations for the pi-field:

$$\left[\phi(\bar{r},t),\,\phi(\bar{r}',t)\right] = 0 = \left[\phi(\bar{r},t),\,\dot{\phi}(\bar{r}',t)\right]$$

and

$$\left[\phi(\bar{r},t),\,\partial_t\phi(\bar{r}',t)\right] = i\,\delta^{(3)}(\bar{r}-\bar{r}')\quad .$$

We do not write out the tachyon commutation relations, which we will not need anyway. Hence, the equation of motion for ϕ is:

$$\left(\partial_t^2 - \nabla^2 + m_\pi^2\right)\phi(x) = \eta(x)$$

The equation of motion for the tachyon field ψ will not be required. For definiteness we wrote it as a Klein-Gordon field with the negative mass squared term displayed explicitly.

The S matrix element we need to calculate is between momentum eigenstates. We assume there is no interference between successive emissions of the Cherenkov π quanta. We make this assumption instead of solving the coupled field equations given by our Hamiltonian theory, since we do not believe that that Hamiltonian field theory has any general significance: we use it only as an effective Hamiltonian theory to generate a few elementary processes.

$$S_{fi} = \langle p_r' k_\pi; \text{out} \,|\, p_r \text{ in} \rangle$$

Using the LSZ reduction scheme ([65] Barton) for the π particle we have:

$$S_{fi} = \delta_{fi} + i \int d^4x \, \frac{e^{ikx}}{V^{1/2}\sqrt{2\omega_k}} \langle p_r' | \eta(x) | p_r \rangle \tag{37}$$

Now using $\eta(x) = e^{iPx}\eta(0)e^{-iPx}$, we get

$$S_{fi} = \delta_{fi} + i \frac{(2\pi)^4 \delta^{(4)}(p - p' - k)}{V^{1/2}\sqrt{2\omega_k}} \langle p' | \eta(0) | p \rangle \tag{38}$$

To evaluate the matrix element of the tachyon scalar current we take the liberty of writing the tachyon field in the form:

$$\psi(y) = \frac{1}{V^{1/2}} \int \frac{d^3k}{\sqrt{2E_r(k)}} \left(e^{-iky} a_k + e^{+iky} \bar{a}_k^+ \right) \tag{39}$$

$\bar{a}_k^+$ creates an anti-tachyon, a_k destroys a tachyon.

Then:

$$\langle p'|\eta(0)|p\rangle = \frac{1}{V}\langle 0|a_{p'}\int d^4y\, K(-y)\int\frac{d^3k'}{\sqrt{2E(k')}}\left[e^{ik'y}a^+_{k'}+e^{-ik'y}\bar{a}_{k'}\right] \int\frac{d^3k''}{\sqrt{2E(k'')}}\left[e^{-ik''y}a_{k''}+e^{+ik''y}\bar{a}^+_{k''}\right]a^+_p|0\rangle \tag{40}$$

where to lowest order in the coupling we neglect the difference between the free and interpolating fields. Since we are looking at the case $p'\neq p$, the contribution is from the term containing:

$$a_{p'}a^+_{k'}\,a_{k''}a^+_p = \left[\delta_{p'k'} \pm a^+_{k'}a_{p'}\right]\left[\delta_{k''p} \pm a^+_p a_{k''}\right]$$

Hence:

$$\langle 0|a_{p'}a^+_{k'}a_{k''}a^+_p|0\rangle = \delta_{p'k'}\,\delta_{k''p}$$

irrespective of whether the tachyons are Bosons or Fermions.

Hence:

$$\langle p'|\eta(0)|p\rangle = \frac{1}{V}\int d^4y\,\frac{e^{i(p'-p)y}K(-y)}{\sqrt{2E(p')}\sqrt{2E(p)}} \equiv \frac{1}{V}\frac{1}{\sqrt{2E(p')}\sqrt{2E(p)}}\tilde{K}(p-p') \tag{41}$$

Now, $\tilde{K}(-p-p')$ is a scalar function, as can be seen by the way in which it is defined. As in the case of the c number tachyon source, we can relate the scalar function in one frame to the function in any other frame, with the defining equation for a scalar function:

$$\tilde{K}'(k') = \tilde{K}(k) \quad ; \quad k' = \Lambda k \tag{42}$$

That the form factor $\tilde{K}(p-p')$ is not just a function of $(p-p')^2$ is argued in appendix (C). As in that appendix, we make use of a transformation of the function $\tilde{K}$ to a standard frame in order to write the dependence of $\tilde{K}(k)$ on the velocity in an explicit manner.

Earlier in this chapter and in appendix (C) we treat the case of a c-number tachyon charge density by transforming to the standard inertial frame in which $E_T = 0$. We argue that $\rho(x)$ in that frame was not a function of Z and that therefore the Fourier transform implied that $k_z = 0$. For a q-number tachyon free to recoil upon emission of the π particle there is no unique frame in which it has zero energy. The frame in which the tachyon initially has zero energy is obviously not the same as the frame in which the tachyon final state has zero energy. In addition, even the tachyon direction, which we call the Z axis, is changed after the emission process because of the recoil.

Actually, what we really need is to find an inertial frame in which the form factor takes on a standard functional form. We now seek a standard inertial frame. In Chapter III equation (31) we derived from conservation of four-momentum: (k^μ is the π four-momentum):

$$E_\tau k^\circ_\pi = \bar{p}_\tau \cdot \bar{k}_\pi + \frac{m_\pi^2}{2} \tag{43}$$

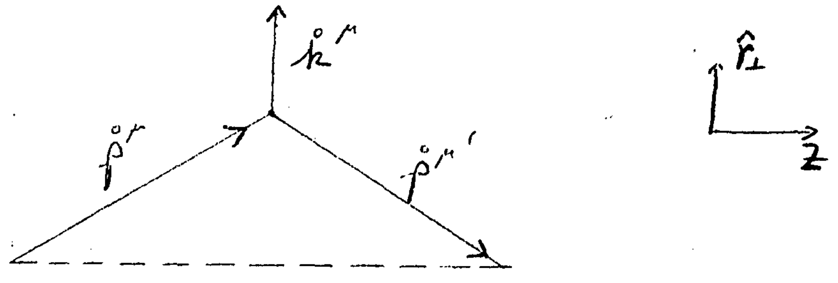
In that derivation the initial tachyon four-momentum appears in the result. In the same way, it is straightforward to derive a relation in which the final tachyon four-momentum appears.

$$E_\tau' k^\circ = \bar{p}_\tau' \cdot \bar{k} - \frac{m_\pi^2}{2} \tag{44}$$

Adding the two we obtain:

$$(E_\tau + E_\tau') k_\circ = \bar{k} \cdot (\bar{p}_\tau' + \bar{p}_\tau) \tag{45}$$

We cannot find a single inertial frame in which both E_τ and E_τ' are zero. However, we can make a Lorentz transformation to an average inertial frame in whihh $\mathring{E}_\tau + \mathring{E}_\tau' = 0$ if we don't use the "reinterpretation principle." The "o" refers to this standard frame: i.e.



Therefore in this frame :

$$|\mathring{E}_\tau| = |\mathring{E}_\tau'|$$

and from equation (45)

$$\mathring{k} \cdot (\mathring{\bar{p}}_\tau' + \mathring{\bar{p}}_\tau) = 0 \tag{46}$$

Now since:

$$\mathring{\bar{p}} = \frac{\mathring{\bar{v}}\,|\mathring{E}|}{c^2}$$

Equation (46) becomes

$$\mathring{\bar{k}} \cdot (\mathring{\bar{v}}' + \mathring{\bar{v}}) = 0$$

We now let $\mathring{\bar{v}} + \mathring{\bar{v}}'$ define the Z axis. Hence $\mathring{k}_z = 0$. If we transform to this frame we can write (cf Appendix (C))

$$\tilde{K}(k^\mu) = \tilde{K}_0(\Lambda^{-1}k^\mu) = \tilde{K}_0(\mathring{k}_x, \mathring{k}_y, \mathring{k}_0) \tag{47}$$

$\tilde{K}_0$ is not a function of $\mathring{k}_z$ since:

$$\mathring{k}_z = (\Lambda^{-1} k)_z \equiv 0$$

To implement this transformation we determine the boost $\bar{u}$ to the standard frame from the condition $\mathring{k}_z = 0$:

We find:

$$\mathring{k}_z = (k_z - k_0 u)\gamma_u = 0 \Rightarrow k_z = k_0 u \tag{48}$$

or:

$$k_0 = \frac{k_z}{u}$$

To find u we divide equation (43) and (44) by $2E_\tau$ and $2E_\tau'$, respectively and add:

$$k_0 = \bar{k} \cdot \left(\frac{\bar{v}_\tau + \bar{v}_\tau'}{2}\right) + \frac{m_\pi^2}{4}\left[\frac{1}{E_\tau} - \frac{1}{E_\tau'}\right] \tag{49}$$

Hence to a good approximation if m_π is small or E_τ is large

$$k_0 \cong \bar{k} \cdot \left(\frac{\bar{v}_\tau + \bar{v}_\tau'}{2}\right) \tag{50}$$

From (48) and (50):

$$\frac{\hat{z}}{u} \cong \frac{\bar{v}_r + \bar{v}_r'}{2} \tag{51}$$

This gives us the magnitude and direction of the boost $\bar{u}$ to the standard frame. We then have from (51)

$$\gamma_u = \frac{1}{\sqrt{1-u^2}} = \frac{\left|\frac{\bar{v}_r + \bar{v}_r'}{2}\right|}{\sqrt{\left|\frac{\bar{v}_r + \bar{v}_r'}{2}\right|^2 - 1}} \tag{52}$$

Hence to find $\overset{\circ}{k}_0$ to insert in (47):

$$\overset{\circ}{k}_0 = (k_0 - \bar{u}\cdot\bar{k})\gamma_u = (\Lambda^{-1}k)_0$$

using (50) and (51):

$$\overset{\circ}{k}_0 = \left[\bar{k}\cdot\left(\frac{\bar{v}_r + \bar{v}_r'}{2}\right) - \frac{\hat{z}}{\left|\frac{\bar{v}_r + \bar{v}_r'}{2}\right|}\cdot\bar{k}\right]\gamma_u$$

Now using (52) for γ_u

$$\overset{\circ}{k}_0 = k_z\sqrt{\left|\frac{\bar{v} + \bar{v}'}{2}\right|^2 - 1} \equiv \frac{k_z}{|\gamma_{\langle v \rangle}|} \tag{53}$$

Inserting this in (47) we obtain the expression for the form factor which explicitly shows the dependence on velocity:

$$\tilde{R}(k^\mu) = \tilde{R}_0\left(k_x, k_y, \frac{k_z}{|\gamma_{\langle v \rangle}|}\right) \tag{54}$$

We have thus isolated the dependence on v in the form factor in the term $\frac{k_z}{|\gamma|}$. This takes care of the "Lorentz expansion" of the source in the Z direction.

In order to get a convenient approximation for $|\gamma_{\langle v\rangle}|$, we note that by repeated application of the arithmetic-geometric mean inequality we can easily show: (for $v>c$)

$$\frac{|\gamma_{v_r}|+|\gamma_{v_r'}|}{2} \geqslant |\gamma_v\gamma_{v'}|^{1/2} \gtrsim |\gamma_{(vv')^{1/2}}| \gtrsim |\gamma_{(\frac{v+v'}{2})}| \tag{55}$$

(Note that we just assumed $\vec{v}, \vec{v}'$ approximately collinear in this frame)

Hence, $|\gamma_v\gamma_{v'}|^{1/2}$ is a better approximation to $|\gamma_{\langle v\rangle}|$ than $|\frac{\gamma_v+\gamma_{v'}}{2}|$. Also, it will be a more convenient approximation.

Therefore, combining this approximation for $|\gamma_{\langle v\rangle}|$ with (54) we obtain

$$\tilde{R}(k) = \tilde{R}_0\left(k_x, k_y, \frac{k_z}{|\gamma_v\gamma_{v'}|^{1/2}}\right) \tag{56}$$

Now using (41) in (38) we have:

$$S_{fi} = \delta_{fi} + i\,(2\pi)^4 \frac{\delta^{(4)}(p-p'-k)\,\tilde{R}(p-p')}{V^{3/2}\sqrt{2\omega_k}\sqrt{2E_r'}\sqrt{2E_r}} \tag{57}$$

With this we can form the transition rate per unit time, ([64] Bjorken and Drell):

$$w_{fi} = \frac{|S_{fi}|^2}{T} = \frac{(2\pi)^4\,\delta^{(4)}(p-p'-k)}{V^2\ 8\omega_k E_r E_r'}\,|\tilde{R}(p-p')|^2$$

where we have used

$$(2\pi)^4\,\delta(0) \approx VT$$

To obtain the rate of energy loss per unit time we multiply w_{fi} by ω_k, the energy of the emitted π particle and by the number of final states of the tachyon and π in the momentum interval $d^3 p_\tau \, d^3 k_\pi$:

$$\frac{V}{(2\pi)^3} d^3 p_\tau' \frac{V}{(2\pi)^3} d^3 k$$

Also using the approximation (56) for $\tilde{K}(k)$ we obtain:

$$\frac{dE}{dt} = \frac{(2\pi)^4}{(2\pi)^6} \iint \frac{\delta^{(4)}(p - p' - k)}{8\,\omega_k E_\tau E_\tau'} \omega_k \left| \tilde{K}_0 \left(k_x, k_y, \frac{k_z}{|\gamma_v \gamma_{v'}|^{1/2}}\right)\right|^2 d^3 p_\tau' \, d^3 k \tag{58}$$

Now look at the integration over the energy conservation part of the delta function, integrated over $d^3 k$.

$$\delta(\omega_k - E_\tau + E_\tau')\, k_\perp \, dk_\perp \, d\phi \, dk_z \tag{59}$$

where we have used cylindrical coordinates. Assume cylindrical symmetry (see Appendix (C)) so that we can do the integral over: $\int d\phi = 2\pi$. Now to do the integral over $k_\perp$ use the relation: [61] Messiah

$$\delta(g(x)) = \sum_n \frac{\delta(x - x_n)}{|g'(x_n)|} \quad ; \quad \begin{matrix} g(x_n) = 0 \\ g'(x_n) \neq 0 \end{matrix} \tag{60}$$

and remember that our analysis of the energy momentum conservation showed that the energy delta function gives, in our approximation (see 50)

$$\delta\left(\omega_k - k_z \left(\frac{v_\tau + v_\tau'}{2}\right)\right) \tag{61}$$

In this case:

$$|g'(x_n)| \to \frac{d\omega}{dk_\perp} = \frac{k_\perp}{\omega_k} \tag{62}$$

a well known relation.

and (61) implies:

$$\omega_k^2 \equiv k_\perp^2 + k_z^2 + m_\pi^2 = k_z\left(\frac{v+v'}{2}\right)^2 \tag{63}$$

$$\Rightarrow k_\perp = \sqrt{\left[\left(\frac{v+v'}{2}\right)^2 - 1\right]k_z^2 - m_\pi^2} \approx \sqrt{\frac{k_z^2}{|\gamma_v \gamma_{v'}|} - m_\pi^2}$$

Hence, we have:

$$\frac{dE}{dt} = \frac{1}{(2\pi)^2}\iint \frac{\delta^{(3)}(\bar{p}-\bar{p}'-\bar{k})}{8E_\gamma E_\gamma'}\left|\tilde{K}_0\left(k_\perp, \frac{k_z}{|\gamma_v\gamma_{v'}|^{1/2}}\right)\right|^2 d^3 p_\gamma' \, 2\pi \, \frac{\delta\left(k_\perp - \sqrt{\frac{k_z^2}{|\gamma_v\gamma_{v'}|} - m_\pi^2}\right) k_\perp \, dk_\perp \, dk_z}{|k_\perp/\omega_k|} \tag{64}$$

Now make the change of variable,

$$k_z' = \frac{k_z}{|\gamma_v \gamma_{v'}|^{1/2}} \tag{65}$$

and use $\omega_k = k_z\left(\frac{v+v'}{2}\right)$ for the factor of ω_k. Then do the integral over $dk_\perp$ and over d^3p_γ' which is trivial because of the $\delta^{(3)}(\bar{p}-\bar{p}'-\bar{k})$.

Therefore we have:

$$\boxed{\frac{dE}{dt} = \frac{v_\gamma + v_\gamma'}{4\pi}\int \frac{dk_z' \, k_z'}{2\,|2m_\gamma|^2}\left|\tilde{K}_0\left(\sqrt{k_z'^2 - m_\pi^2}, k_z'\right)\right|^2} \tag{66}$$

To obtain this we used

$$k_z\, dk_z \rightarrow |\gamma_v \gamma_{v'}|\, k_z' dk_z' = \frac{E_\tau E_\tau'}{|m_\tau|^2} k_z' dk_z' \tag{67}$$

and $\overline{\frac{v_\tau + v_\tau'}{2}}$ is an average velocity of the tachyon, averaged over possible final velocities. Strictly speaking, this factor should be within the integral sign since v_τ' is a function of k_z. The bar above $\overline{\frac{v_\tau+v_\tau'}{2}}$ indicates that this is the definition of the average. We assume recoil is weak enough to make this meaningful, i.e, $\omega_{max} << E_\tau$. This is not Lorentz invariant of course. In fact, we assume that the tachyon does not pair annihilate in the observor's frame. Hence:

$$\overline{v_\tau} \equiv \overline{\frac{v_\tau + v_\tau'}{2}} \simeq v_\tau$$

Now the integral in (30) is independent of v_τ and is just equal to some constant. $\tilde{K}_0$ contains the necessary cutoff or ω_{max} so $\frac{dE}{dt}$ is finite, according to the appendix (C). Hence, we find on the average:

$$\frac{dE}{dt} \propto \overline{v_\tau}$$

as predicted. Following equation (66) the same analysis applies as is found for the c-number, non-recoiling, superluminal source.

We saw that both the classical c-number and the quantum derivation, the latter with certain approximations, yielded the same form of the energy loss. The further analysis follows from this result and applies to both cases. Each derivation, for $m_\pi \neq 0$, yields the existence of a minimum value of the k vector, k_{min} . To see the consequence of this we need to look at a particular model for the form factor or charge density. We will use the notation of the c-number section but $\rho(k)$ can be thought of as equivalent to $\frac{\tilde{K}}{(2m_\pi)}$ (Compare equations (36) and (66).)

As a simple example of ρ , we look at a sphere in k space: i.e. $\rho_0(k_\perp, k_z) = \rho\left(\sqrt{k_\perp^2 + k_z^2}\right) \quad = \rho\left(\sqrt{2k_z'^2 - m_\pi^2}\right)$ (from our value for $k_\perp$); $\rho(k) = g\,\Theta(-k + k_{max})$ is a possibility, where $k_{max} \sim \frac{1}{r_0}$ or m_π, etc.

Hence:
$$\frac{dE}{dt} = \frac{g^2}{4\pi} v \left.\frac{k_z'^2}{2}\right|_{m_\pi}^{\sqrt{\frac{k_{max}^2 + m_\pi^2}{2}}} = \frac{v g^2}{4\pi} \frac{1}{2}\left(\frac{k_{max}^2 + m_\pi^2}{2} - m_\pi^2\right)$$

$$\therefore \quad \frac{dE}{dt} = \frac{v g^2}{16\pi}\left(k_{max}^2 - m_\pi^2\right) \qquad (67)$$

Hence, we see from (67) that if $m_\pi \geq k_{max}$ there will be no generalized Cherenkov radiation. In general, there won't be a sharp cutoff in k space. There are three possibilities.

a) If k_o represents the approximate extent of appreciable values of ρ in k space, then if $m_\pi \gg k_0$ there will be no generalized Cherenkov radiation.

b) If $k_0 \gg m_\pi$, the radiation will be uninhibited by the cutoff.

c) If $k_0 \approx m_\pi$ there will be only a small amount of radiation.

We see that there is a possibility for tachyons to be strongly coupled to a massive field but with a cutoff such that the generalized Cherenkov radiation might be less than that produced by a coupling to the electromagnetic field. There might be no generalized Cherenkov radiation at all. Hence, tachyons might be produced by strong interactions and not lose energy by massive Cherenkov radiation.

In order to see the characteristics with respect to angle and energy of the emitted particles, multiply equation (31) by $\hbar\omega_k$ and a delta function picking out the angle or E_π one is interested in, and then integrate over $\bar{k}$. There is a one to one mapping of k_z onto angles or energy and we can use the relations:

$$\sqrt{\bar{k}^2+m_\pi^2} = \omega = \bar{k}\cdot\bar{v} \equiv k_z v = |\bar{k}|\, v \cos\theta \tag{68}$$

with a little algebra to find: I ≡ intensity of radiation of π's

$$\frac{dI}{dk_z} = v\frac{dI}{d\omega} = v\frac{d\theta}{d\omega}\frac{dI}{d\theta} \tag{69}$$

and since $v_\pi = \dfrac{1}{v_\tau \cos\theta}$

$$\omega_\pi = \frac{m_\pi v_\tau \cos\theta}{\sqrt{v_\tau^2\cos^2\theta - 1}}\text{, and} \tag{70}$$

$$\frac{d\omega}{d\theta} = \frac{m_\pi v_\tau \sin\theta}{(v^2\cos^2\theta-1)^{3/2}} \tag{71}$$

Now we can integrate over $\bar{k}_\perp$ just as previously in this chapter if we assume cylindrical symmetry. Therefore

$$\frac{d^2E}{dk_z\,dt} = \frac{v_\tau\left(\frac{v_\tau^2}{c^2}-1\right)}{4\pi}\, k_z \left|\rho\left(\sqrt{(v^2-1)k_z^2-m_\pi^2},\ \frac{k_z}{|\gamma|}\right)\right|^2 \tag{72}$$

For example, the rate of energy radiated at an angle θ is (use $k_z = \frac{\omega}{v_\tau}$ and (70)

$$\frac{d^2E}{dt\,d\theta} = \frac{d^2E}{dt\,dk_z} \cdot \frac{1}{v_r}\frac{d\omega}{d\theta} \tag{73}$$

and therefore: $(d\Omega = 2\pi \sin\theta\, d\theta)$

$$\frac{dI}{d\Omega} = \frac{v_r\left(\frac{v_r^2}{c^2}-1\right)}{8\pi^2}\, \frac{m_\pi^2 \cos\theta}{\left(\frac{v^2}{c^2}\cos^2\theta - 1\right)^2} \left| \rho\left(\sqrt{(v^2-1)k_z^2 - m_\pi^2},\ \frac{k_z}{|\gamma|}\right)\right|^2 \tag{74}$$

In this equation the expression for k_z in terms of θ, obtained from dividing (70) by v, should be used in the argument of ρ so that the RHS is a function of θ. Note that as $m_\pi \to 0$ this expression goes to zero except when the denominator is 0 at $\frac{v^2}{c^2}\cos^2\theta = 1$ which is the usual electromagnetic Cherenkov angle for light if the index of refraction is one.

When Sommerfeld calculated the radiation for an electrically charged configuration with v > c, he found an infinite rate of energy loss from an infinitesimally thin spherical shell, [o4] Sommerfeld. We show in appendix (F) that this is also true in the case of generalized Cherenkov radiation of particles with a finite mass.

In appendix (D) we consider in more detail the various types of radiation encountered in creating a particle which exceeds the velocity of light. In addition to the generalized Cherenkov radiation which we considered in Chapter IV, we find an additional radiated portion which corresponds to destroying the charge at rest and immediately creating the superluminal particle. One portion of the radiation is found to be independent of the lifetime of the tachyon for large T. It corresponds to "prompt" radiation analogous to the beta decay inner bremsstrahlung produced by accelerating (or creating) a charge from rest to a final velocity, [62] Jackson This prompt radiation appears here twice; as the radiation that would be obtained if the charge were created, and then as the radiation obtained as if the charge were destroyed. Another portion, interference radiation, also not Cherenkov radiation, is seen to oscillate as a function of T.

In appendix E we consider the interpolating field in order to gain insight into how the radiation develops.

In the above calculations of the various properties of generalized Cherenkov radiation it was assumed that the velocity of the radiating particle was approximately <u>constant</u>. This makes it easier to determine the direction and existence of the radiation through a Huygens construction and geometric considerations. However, according to our derivation of hyperbolic motion in Chapter III, the tachyon will be constantly accelerated as it emits Cherenkov radiation. The influence of this acceleration on the direction of the Cherenkov radiation is obviously of some importance for the theory. From an experimental standpoint, it is necessary to know the characteristics of Cherenkov radiation from a recoiling tachyonic source.

To attack this problem, in appendix G we deduce the envelope of the field generated by a tachyon in hyperbolic motion.

motion. Using Leibnitz's method for the envelope of a family of curves we show that the radiation in a particular case (at the time when $v_r = \infty$) is a half circle rotated about the direction of propagation. Such a result is of importance in designing a successful experiment to detect tachyon produced Cherenkov radiation; radiation which is found to be focused on a ring of radius g^{-1}.

It is found that for this motion, at least, the derivations of cos θ in Chapters III and IV give the correct results at each instant.

Chapter V

Application of Results

We now comment on some of the experiments which have been carried out. On the basis of our results, we question the interpretation which has been given to the findings reviewed in Chapter II.

In the second part of this Chapter we describe realistic approaches to detecting faster-than-light particles suggested by our calculations.

In the first experiments which were carried out, Alvager and Erman [65] assumed that tachyons have an electric charge and are acted on by external fields, although not subject to Cherenkov radiation. We have shown that, indeed, this is possible for coupling to a massive field. However, for the electromagnetic field the prohibition of radiation does not seem justified since we found no minimum wave vector of emitted radiation if $m_\pi = 0$.

The experiments based on a search for electromagnetic Cherenkov radiation, were also negative. Alväger and Kreisler [68] and Davis, Alväger and Kreisler [69] used an electric field, and Bartlett and Lahana [72] used a magnetic field, to accelerate electrically and magnetically charged tachyons, respectively. The equation they use for the rate of energy loss due to Cherenkov radiation is based on the usual Cherenkov energy loss in a medium.

$$\frac{dE}{dx} = -\frac{4\pi^2 z^2 e^2}{c^2}\int\left(1-\frac{c^2}{v^2 n^2}\right)\nu\, d\nu$$

They take the index of refraction, n, equal to one for a vacuum and use the tachyon energy as the cutoff in the integral in order to get a finite value. The result they obtain is:

$$\frac{dE}{dx} = -\frac{z^2 e^2 m^2 E^2}{2\hbar^2 \bar{p}^2}$$

which is not Lorentz covariant. In fact, as we indicated in Chapter III, the rate of energy loss would be expected to be greater than the rate given by this. When we put in some reasonable parameters in Chapter III, we found that the expected range would be 10^{-9} cm.

The field used by AK was 3 KV. This would not have had much effect if the parameters were approximately those of an electron. If the field Cherenkov radiated were more strongly coupled then the range would be even less. The only field more weakly coupled that has been considered, namely gravitation, couldn't have been detected although the range would be much greater.

Baltay, Feinberg, et al [70] claim that their missing mass experiment has the advantage over other approaches of being "insensitive to unsolved problems of the interaction of tachyons with matter or their propagation through space." According to our derivation of generalized Cherenkov radiation though, it is quite possible that the tachyons could not propagate far enough to be "missing". If indeed they are strongly interacting, they might lose all their energy over extremely short distances. The products of this radiation of hadrons would appear in the bubble chambers displaced a microscopic distance from the other reaction products. Hence, we conclude that their null result applies only to those spin 0 tachyons whose massive Cherenkov radiation is inhibited according to the possibility derived in our theory in Chapter IV.

In the cosmic ray search by Clay and Crouch [74] the extensive air shower (EAS) with which tachyons might have been produced were assumed to originate at heights between 20 km. and 400 m. The tachyon interaction with the scintillator may have been direct or through the generation of secondary particles. Hence, the tachyons accessible to the experiment need not have been electrically charged. The signal was then fed into a transient recorder which was examined when triggered by the subsequent arrival of an EAS. The presence in the transient recorder of a precursor presumably indicated something whihh traveled faster than the EAS, which itself is supposed to go at essentially the speed of light.

According to oureestimate in Chapter III of the range-energy relation for electrically charged tachyons, if the tachyon produced with the EAS had an energy of the order of 10^{15}eV its range would be only about 10 meters because of the loss of energy through Cherenkov radiation. Of course, the parameters relevant to the tachyon could be quite different from those we assumed. It is possible that the parameters are such that the tachyon range is larger than our estimate, in fact large enough for a 10^{15}eV tachyon to travel 20 km and yet not be incansistent with the null result of AK and DAK. This is easily seen from the constancy of dE/dx in III-(46). Hence, a 10^3 fold increase in range at 10^{15}eV would also correspond to a 10^3 fold increase in the range of any tachyon produced in the experiments seeking Cherenkov radiation. By the considerations in Chapter III on the order of magnitude of the range, such an increase would not have changed the outcome of those investigations.

However, barring this possibility for the moment, we assume that electrically charged tachyons of that energy could not have traveled so far. We now look at the possibility that the purported tachyons were not electrically charged, but were strongly charged. If in fact, the tachyons are strongly charged then according to the results of Chapter IV, they may be subject to a generalized Cherenkov radiation and have an even shorter range than just discussed. On the other hand, we also showed in Chapter IV that the existence of k_{min} creates the possibility that the generalized Cherenkov radiation might be suppressed by the tachyon form factor, thus greatly increasing the tachyon's range. If the latter possibility is in fact the case, then the strongly interacting tachyons could have been detected by Clay and Crouch.

The latter case is however, inconsistent with the null result of Baltay, Feinberg et al which we interpreted above as excluding the existence of very long range tachyons with suppressed massive Cherenkov radiation. Very long range tachyons would have been detectable by the EAS and the missing mass experiments; and on the other hand very short range tachyons would not have been detected by either. Hence, we ask if it is possible that the generalized Cherenkov radiation is in the intermediate range found in Chapter IV, according to which the radiation is only partially inhibited by the existence of k_{min} and the form factor. Then the range might be long enough at 10^{15}eV to be detected by Clay and Crouch and yet short enough not to be detected by Baltay, Feinberg et al. Again the constancy of dE/dx which we found in Chapter III allows us to answer this. The linearity thus implied between x and E and the intersection with the origin means that a range of 1 km at 10^{15}eV extrapolates to 10^{-4}cm. at 1 MeV.

Hence, we conclude that these experiments can be made consistent with each other by assuming that the generalized Cherenkov radiation which we derived in Chapter IV exists, and that the inhibitory mechanism which we found also is present.

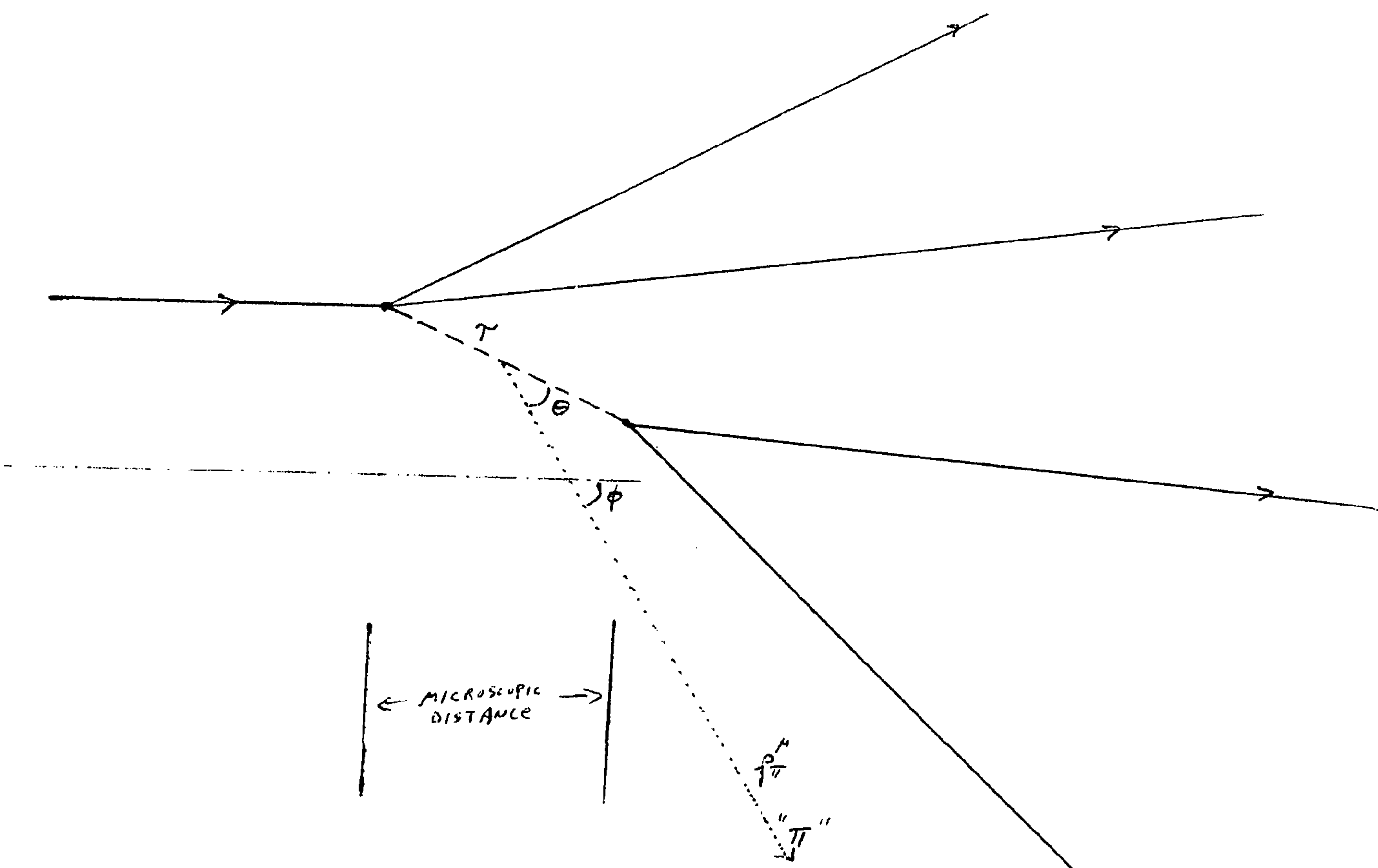
We had derived in Chapter III that a tachyon emitting electromagnetic Cherenkov radiation would lose all its energy over extremely short distances, subject of course to the parameters one postulates for the tachyon. Hence, as compared with equally energetic ordinary particles emitting Cherenkov radiation, one would expect a short, extremely intense burst of radiation. As we have seen, such short range frustrates attempts to detect them.

However, according to equation IV-17 for ω_{max} for electromagnetic Cherenkov radiation in a non-dispersive medium the radiation will be inhibited because of the conservation laws. Althoggh the equation is usually thought of as applying to electrons in a dielectric, we have derived it on very general grounds; hence it should be applicable to a tachyon in a medium also. Dispersion must of course, be taken into account for any particular medium. For a tachyon, the radiation will again be present at frequencies sufficiently high that $n(\omega) \to 1$. But then the tachyon form factor will begin to inhibit it. Still, this presents the possibility for experimentalists to retard the energy loss of the tachyon so that an experiment such as that of AK might become practical by creating the tachyon in a medium which has very large n which extends to very high frequencies. One would then have to look for the ectromagnetic Cherenkov radiation at those frequencies for which $n(\omega) \simeq 1$, or for frequencies below ω_{max}. In general, there will be required a careful analysis of $\omega_{max}(n)$ since n is a function of ω.

The relation between angle and energy we derived, and the formula for $dI/d\Omega$, IV-74, could possibly be exploited to detect the presence of a very short lived tachyon. Investigating bubble chamber photographs, if a correlation between incident particle and emitted products is found to indicate our derived relations, this may serve as evidence of tachyons emitting generalized Cherenkov radiation. See figure (10) which illustrates a typical reaction with the parameters involved. A statistical analysis of the relation of angle and energy of the "π" would have to be carried out for many reactions. Of

III-C-6

POSSIBLE BUBBLE CHAMBER TACHYON SEARCH



$$\left(p_\pi^\mu,\ \theta\right)$$

$$\cos\theta = \frac{c^2}{v_\tau v_\pi}$$

ϕ AND E_π ALSO CORRELATED
(∴ don't need v_τ)

figure 1D

course, if one knew v_τ and v_π for the cos θ equation this would not be necessary since we would already know whether or not $v > c$. However, for Cherenkov radiation we have the characteristic feature that greater angles will in general be associated with greater energy particles (both the initial incident one and the Cherenkov radiated one.

If, on the other hand, the tachyon is extremely energetic, having been created by a cosmic ray for example, its path may be of detectible length in the bubble chamber. Then the generalized Cherenkov radiation might be directly observed as many tracks originating along a common (invisible) straight line. The straight line could then be inferred from the origins of the tracks. The angles would be measured and compared with the energies of the radiated particles. A plot of cos θ versus $1/v_\pi$ for all of the tracks could then be made. If this were found to be a straight line, it would be an indication of the presence of a tachyon. The inverse of the slope of this line would then yield the tachyon velocity. The use of the equation (IV-10)

$$\cos \theta = \frac{c^2}{v_\tau \; v_\pi}$$

implies that we are assuming relatively little recoil. A more complicated analysis would be made if the plot were not a straight line, using the exact relation (III (32)).

$$\cos \theta = \frac{c^2}{v_\pi \; v_\tau} - \frac{m^2}{2|\vec{p}_\tau| \, |\vec{p}_\pi|}$$

The hypothetical tachyon line in the bubble chamber would also not be a straight line in this recoiling case and the origins of the visible tracks would have to be connected by a zig-zag line.

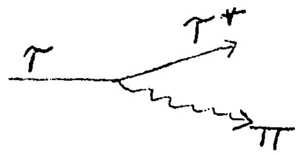
Another experiment might consist in looking for the focussing of the Cherenkov radiation associated with the hyperbolic motion derived in appendix (G). Probably electromagnetic radiation from a tachyon produced by cosmic rays would be most likely. Depending on the value of g^{-1} one might have the detector beyond the point of focus (the most likely case) and hence note the "virtual" image of the ring.

Appendix A

A New Derivation of the Kinematics For More General Conditions And Related Phenomena

Analysis is made of the constraints which conservation of energy and momentum impose on the emission of radiation by various types of sources. We consider the question from a general viewpoint so that the results are not limited to superluminal motion in a vacuum nor to electromagnetic radiation.

We continue to denote the radiating particle by τ but will indicate where the results are valid for tachyons only, or for more general situations. Allowing for the possibility of a change in the internal excitation of the radiating particle, we consider the situation



The square of the four-momentum is given by:

$$p_\mu^\tau(t)\, p_\mu^\tau(t) = -m_\tau^2(t) \tag{1}$$

Since we assume that the particle denoted by τ retains its identity although possibly changing its state during the event described, the equation is satisfied both before and after the event. The mass and momentum are shown as a function of a parameter t which may be thought of as the time in any inertial frame. Taking differentials:

$$p_\mu^\tau(t)\, dp_\mu^\tau(t) = -m_\tau(t)\, dm_\tau(t) \tag{2}$$

We assume that there is no interference between two different emissions. Now, by conservation of four momentum, we have at any time either before, during, or after the emission process:

$$p_T^{\mu}(t) + p_{\pi}^{\mu}(t) = p_T^{\mu}(0) \tag{3}$$

and

$$d\,p_T^{\mu}(t) + d\,p_{\pi}^{\mu}(t) = 0 \tag{4}$$

Using (3) and (4) in equation (2) we obtain:

$$\left(p_{\mu}^{T}(0) - p_{\mu}^{\pi}(t)\right) d p_{\mu}^{\pi}(t) = + m_T\, d m_T \tag{5}$$

And integrating from before the emission ($t = 0$) to a time T which is after the emission of the π particle:

$$p_{\mu}^{T}(0)\, p_{\mu}^{\pi}(T) - \frac{p_{\mu}^{\pi}(T)\, p_{\mu}^{\pi}(T)}{2} = -\frac{1}{2}\left[m_T^2(0) - m_T^2(T)\right] \tag{6}$$

We obviously used $p_{\mu}^{\pi}(0) = 0$ in equation (6). Now using the relation:

$$p_{\mu}^{\pi}(T)\, p_{\mu}^{\pi}(T) = m_{\pi}^2 \tag{7}$$

and the approximation:

$$\Delta m_T = m_T(T) - m_T(0) << m_T \tag{8}$$

We obtain the very general result:

$$p_{\mu}^{T}(0)\, p_{\mu}^{\pi}(T) - \frac{m_{\pi}^2}{2} \cong + m_T \Delta m_T \tag{9}$$

The internal excitation energy difference between the final and the initial state is:

$$\Delta U = \Delta m_T \cdot c^2 \tag{10}$$

Defining θ as the angle between the direction of the emitted particle and the initial tachyon we obtain from equation (9):

$$E^{T}_{(o)} E^{\pi}_{(T)} - |\bar{p}^{T}_{(o)}||\bar{p}^{\pi}_{(T)}| \cos\theta - \frac{m_\pi^2}{2} \cong + m_\tau \Delta m_\tau \qquad (11)$$

Solving for the direction of emitted radiation $\cos\theta$ and using $\bar{v}_g = \bar{p}c^2/E$:

$$\boxed{\cos\theta = \frac{c^2}{v_\tau v_\pi} - \frac{m_\pi^2}{2|\bar{p}^{T}||\bar{p}^{\pi}|} + \frac{m_\tau \Delta m_\tau}{|\bar{p}_{(o)}| \, |\bar{p}^{\pi}_{(T)}|}} \qquad (12)$$

This equation applies to a number of different phenomena. It is interesting to see how these are related to superluminal motion. That such a relation exists is due to the fact that only for superluminal motion in a vacuum or in a medium, can a particle emit another particle without changing it's "internal" state. For this reason the kinetic energy of the particle has an absolute and not just a relative significance. Hence, it is available to enter in processes even when the particle is isolated from other bodies.

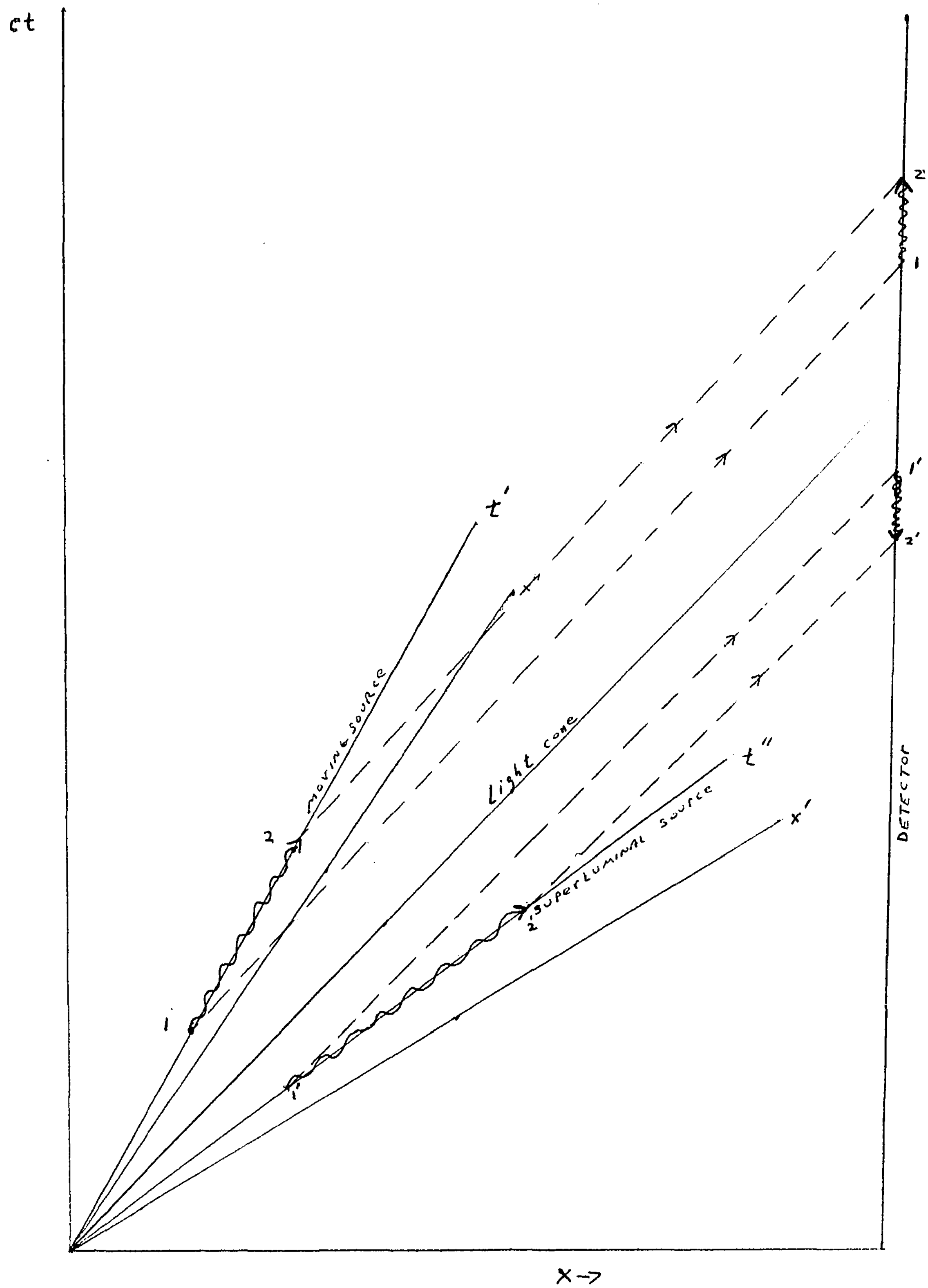
a) In the equation, if $\Delta U < 0$ one has the decay of a system or of an elementary particle. If $m_\pi = 0$, this gives the Doppler shift if $v < c$. In case $m_\pi \neq 0$, one might call this a generalized Doppler shift.

b) If $\Delta U > 0$, i.e. the particle jumps to an excited state upon emitting a particle, then the velocity must be greater than c/n in a medium. The requisite energy comes from a decrease in the kinetic energy. For the case $m = 0$ this is called the anomalous Doppler effect [60] Tamm, [60] Ginzburg. In effect, a negative energy difference or frequency is "Doppler shifted" to a positive energy. The radiation is emitted within the forward Cherenkov cone.

In a vacuum, if $v > c$ then $\Delta\nu$ can only be zero or imaginary. Figure (11) is a Minkowski diagram showing one subluminal and one superluminal source of electromagnetic radiation. The dotted lines indicate the propagation of the radiation parallel to the light cone to the detector world line. The latter is assumed to be at rest. The arrows on the detector time axis show the reversal in time ordering and therefore in frequency of the radiation from the superluminal source. If one ignored the x' and x'' axes this figure would also describe the ordinary anomalous Doppler effect in a medium with c replaced by c/n. If m_π is not zero then this case might be called the "generalized Anomalous Doppler effect". This would describe emission of a massive quantum within the forward Cherenkov cone, while at the same time the tachyon jumps into an excited state.

c) If $\Delta\nu = 0$ then v must be greater than the relevant limiting velocity, and we have either ordinary or massive Cherenkov radiation. The latter is the situation we focus on in this thesis.

It is interesting to note that a number of strange properties ascribed to tachyons can be understood on the basis of equations (9) and (10). In addition it can be shown that these have their counterparts in a medium. For example, Feinberg (1967, appendix A) analyzes the emission and absorption of a tachyon by an atom in two inertial



THE ANOMALOUS DOPPLER EFFECT

figure 11

frames. This has been described in Chapter II.

The counterpart of this process in a medium with ordinary particles is the following. Initially, an atom is at rest in the medium. It is in its ground state and then absorbs a photon. Next, consider an atom boosted in velocity such that $v > \frac{c}{n}$. This atom, although initially in its ground s state, can emit a photon and jump to an excited state at the same time losing kinetic energy. This is the "anomalous Doppler effect".

The latter, "active", transformation cannot be replaced by a "passive" transformation in which the observor undergoes the boost. This is because the medium does not satisfy the same principle of relativity of inertial frames which the vacuum does.

Appendix B

An Alternate Interpretation Of Space-Like Four Momentum

In order to visualize more clearly the implications of negative mass squared, i.e. spacelike four-momentum, we consider the following related problem. A spacelike electric current is ~~typified~~ by current flowing throggh a wire or in an electrolyte. The charge density can be zero but there still exists nonzero current. For the sake of symmetry, imagine a positive charge density flowing to the right and a negative charge density flowing to the left. In the initial frame of reference, the net charge density is zero.

$$j^{\mu} = (0, \vec{j}) \tag{1}$$

Now, a Lorentz boost transformation in the direction of $\vec{j}$ will result in a negative charge density because there will be a greater Lorentz contraction of the line of negative charges than of the line of positive charges. The usual Lorentz transformation is:

$$j^{\mu'} = \left(-\frac{\vec{j}\cdot\vec{u}}{c^2}, \vec{j}\right)\gamma \tag{2}$$

Of course the current is still space-like and:

$$j^{\mu'} j^{\mu'} = -\vec{j}\cdot\vec{j} < 0 \tag{3}$$

If we were to view j as a convection current produced by the motion of a single particle, rather than by the motion of a nonlocalized many body system, weuwould calculate the velocity of the charge flow as:

$$\bar{v}' = \frac{\vec{j}'}{\rho'} = \frac{\vec{j}\,\gamma}{-\vec{j}\cdot\frac{\bar{u}}{c^2}\gamma} \tag{4}$$

Hence:

$$\bar{v}' = \frac{\hat{j}}{-\hat{j}\cdot\bar{u}/c^2} \tag{4a}$$

This is to be compared with the velocity transformation of a tachyon, starting from a frame in which it has infinite velocity in the plus x direction. We make a Lorentz transformation of velocity $\bar{u}$ of this zero energy tachyon (i.e. $v = \infty \leftrightarrow E = 0$)

$$\bar{v}' = \frac{\bar{v}-\bar{u}}{1-\frac{\bar{u}\cdot\bar{v}}{c^2}} \xrightarrow[\bar{v}\to\infty]{} \frac{\hat{v}}{-\frac{\bar{u}\cdot\hat{v}}{c^2}} \tag{5}$$

Comparing equations (4), ~~4~~5), and IV-(9) (the Cherenkov condition), we recognize the transform condition between an inertial frame in which a tachyon has infinite velocity and one in which a tachyon velocity is equal to $\bar{v}'$. In equation (4), we see that since the Lorentz transformation velocity is less than c, the effective current velocity for a space-like current is always greater than c. Also note that if $\bar{u}\to 0$, $\bar{v}\to\infty$. However, such a spacelike current is quite common, in fact the usual case. We certainly do not ascribe any significance to this one particle velocity which becomes ∞ when $\rho\to 0$. Because of the many particle nature of the current, we can not say that the current is a charge density times a physically significant velocity. Similarly, for tachyons (which also seem to be very nonlocalizable ([69] Peres) perhaps there is no justification for saying that the current of energy (i.e. the momentum) is the energy times a physically significant velocity, i.e. $\bar{p} = \frac{E\,\bar{v}}{c^2}$. Perhaps the fact that $\frac{|\bar{p}|c^2}{E} > c$ has as little significance as does an analogous fact in the above example for electric current.

Note that in the tachyon case $\Delta t'$ becomes negative when E_τ becomes negative under a Lorentz transformation. But negative $\Delta t'$ is based on the "reasonable" assumption that $v_\tau = \frac{|\bar{p}|c^2}{E} > c$, and v_τ is the velocity used in $\Delta t' = \Delta t(1 - \frac{u \cdot v_\tau}{c^2})\gamma$. If we were to reject this idea of $v > c$ as we did with currents, then Δt would not change sign (it wouldn't even exist), and we would not use the reinterpretation principle of BDS. We would be forced to deal with the negative energies in some other way.

This leads one to consider the possibility of considering the spacelike four-momentum solutions of the $m^2 < 0$ Klein-Gordon equation as positive energy flowing in the direction of $\bar{p}$ and negative energy flowing forward in time in the $(-)\bar{p}$ direction. Hence, in a certain inertial frame, the energy density can be 0 but we still have a net flow of energy; i.e. $\bar{p} \neq 0$. This previously was interpreted as ∞ velocity. In the usual interpretation of $E = 0$, $v = \infty$, apparently no energy is transported because $\Delta t = 0$. However, with the electric current system analyzed above, we have net transport of charge even though the charge density at intermediate points is always zero, in one inertial frame. If we reject the single particle superluminal velocity interpretation of spacelike four-momentum, then in a frame in which $E_\tau = 0$ the velocity is not ∞ and the process need not be instantaneous. Then the duration of nonzero $\bar{p}$ can be finite and hence if we interpret this as positive and negative energies flowing in opposite directions, we have net energy transfer.

The present visualization of $m^2 < 0$ Klein-Gordon solutions then leads to retention of their negative energy and nonlocalizability aspects, which are similar to those of the electric cu current. The usual reason for rejecting the negative energies (propagating forward in time) is the instability that arises. However, analysis of the $m^2<0$ Klein-Gordon equation has shown that the solutions may indeed be nonlocalizable and/or unstable ([69] Aharonov, Komar and Susskind).

We thus have a picture of energy flow without the necessity of a net energy density in intermediate regions, similar to the transport of charge with spacelike electric current.

We need now a conserved quantum number in order to keep the spacelike field from "blowing up".

The conclusion we may draw is that the negative mass squared Klein-Gordon equation may describe physical phenomena without representing superluminal signals.

C

Charge Density and Form Factor

We have assumed that the source is a scalar with velocity greater than c. Both for a c number source with prescribed constant velocity, and for a quantum source free to recoil upon emission of the π particle, there are important differences as compared with the treatment of ordinary form factors. The "internal" state of the tachyon is assumed to be unchanged by the individual emission process.

Generally, the form factors are assumed to depend on the invariant quantities which can be formed from the four-momenta at the vertex p^{μ}, $p^{\mu'}$, k^{μ}, ([66] Gasiorowicz).

Figure (12a) shows a π exchange between a proton and another particle. The shaded bubble at the p-π vertex represents the proton form factor. Figure (12b) depicts emission of a real pi particle by a tachyon. In figure (12a) p^{μ} and $p^{\mu'}$ are assumed on the mass shell. Hence the form factor is a function only of $t \equiv (k)^2$. Since $v < c$, k cannot be on the mass shell. However, for the situation in which $v > c$ (case b), we may put k on the mass shell also. Then the form factor would be a function of,

$$t \equiv (p' - p)^2 = -k^2 = m_\pi^2 \tag{1}$$

We see, since this is constant, that the Cherenkov radiation would not be cut off by the form factor and would lead to a divergent energy loss because the "volume" of the mass hyperboloid for the pion is infinite.

To deal with this problem, and to see what kind of function a tachyon form factor would be, we must first analyse the

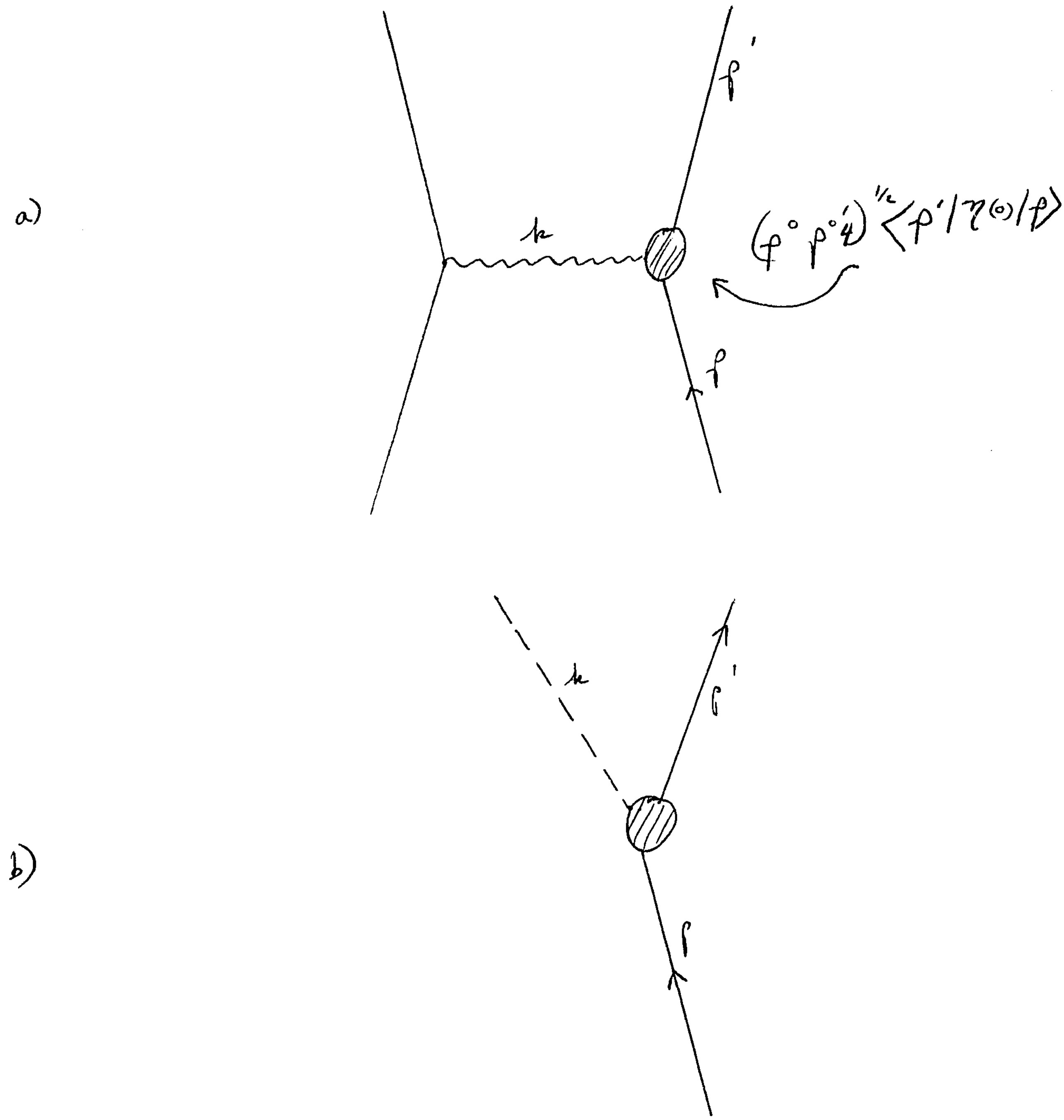


figure 12

situation for $v < c$ more carefully.

Sometimes the phrase "invariant function" is used where the phrase "scalar function" might be employed (see [63] Schrödinger, for example). However, the designation "invariant function" is also used in a more restricted sense. If we have a Lorentz transformation (Λ, a) such that $x' = \Lambda x + a$ then a "scalar" function is one that satisfies:

$$F'(x') = F(x) \tag{2}$$

That is, the transformed function, evaluated at the transformed point, has the same value as the original function at the original point. Compare this with the concept of an "invariant" function as used in the following sense:

$$F(\Lambda x) = F(x) \tag{3}$$

That is, the "original" function evaluated at the transformed point has the same value as it has at the untransformed point, if the transformation is a homogeneous Lorentz transformation. In the former case the function F can be arbitrary; F' is determined by F and the transformation. In the latter case F can be an arbitrary function of x^2 (and sign (x) if x is timelike) which are the invariants formed from x^μ. But on any one branch of the hyperboloid x^2 = constant, F is constant. Well known examples of invariant functions are the functions $\Delta, \Delta^+, \Delta^-, \Delta_F$, formed from the commutator and other vacuum expectation values of a "scalar" field ([61] Schweber). To avoid confusion, we will use "invariant" only in the latter sense, distinguishing it from "scalar".

The form factors for ordinary particles are usually taken to be "invariant functions" without much comment. We need to examine this more carefully. These particles are taken to be "elementary systems" whose states form a representation space for an irreducible representation of the inhomogeneous Lorentz group. The mass and spin, (m, s) label the particular representation and this determines the kinematic description of the free, noninteracting particle ([61] Schweber).

We investigate the symmetry of the form factor in the rest frame of the scalar particle. It is then natural to say that the form factor will be invariant under the subgroup of the inhomogeneous Lorentz group which leaves the wave-vector p_r^{μ} invariant, i.e. the little group. In the particle rest frame we have:

$$p^{\mu} = (m,0,0,0)$$

Therefore, the little group is O(3)--the group of rotations in ordinary three dimensional space ([62] Hamermesh). In general, the irreducible representations of this little group are denoted $D^{(j)}$; we take $j = 0$ for a scalar particle. The form factor will therefore have the symmetry of the little group O(3) if it is a function of the invariant $\bar{r}^2 = x^2+y^2+z^2$ or of $\bar{k}^2$ only.

For a time varying charge the idea of spherical symmetry refers only to a measurement of the charge distribution on a three dimensional hyperplane perpendicular to the time axis in the rest frame of the "center" of the charge. We may then require as an additional assumption that it is static and therefore not a function of t.

In this standard frame the form factor F, by virtue of its supposed symmetry, has no additional four vectors associated with it. The invariant form factor F can then only be a function of the invariant $t \equiv k^2$, since all other invariants formed from p^μ, $p^{\mu'}$, k^μ can be written in terms of $t = k^2$ when $(p^\mu)^2 = m_T^2 = (p^{\mu'})^2$.

If the form factor did not possess this symmetry (O(3)) there would be other intrinsic four vectors, n^μ, to form invariants with p^μ, $p^{\mu'}$, k^μ. F would still be an invariant function of these however.

If the symmetry in the z direction were broken by some interaction with an <u>external</u> system we would find that F_0 would then be a function of x^2+y^2, z. There would then be an additional four-vector m^μ, of which the F_0 would be a function. The little group of the free particle O(3) would then no longer apply, and the form factor would no longer be an invariant function since an external influence would provide a preferred direction. In addition, if the world line were finite, ρ_0 (the charge) would have to be multiplied by $[\theta(t-t_1) - \theta(t-t_2)]$. The charge density ρ_0 would still be static for intermediate times.

For ordinary particles where the "in" and "out" states can be considered to be asymptotically free, these comments are irrelevant since the deviation from a true invariant function is infinitesimal. We only make these distinctions for later comparison with the form factor and charge density of a tachyon whose world line is finite.

We now proceed to investigate the dependence of the tachyon form factor on the variables x^μ. First we look at the free (infinitely long world line) spin 0 tachyon. This is the only finite dimensional unitary representation of the inhomogeneous Lorentz group for spacelike four-momentum ([58] Shirokov). The standard frame for investigating the little group in this case is one in which the energy is zero:

$$p^\mu = (0;0,0,m)$$

The little group is the subgroup which leaves p^μ invariant. It is therefore the non-compact group SO(2,1) of rotations in pseudo-euclidian space with two space (x,y) and one time coordinate. It contains rotations about the z axis and Lorentz boost transformations in the x-y plane. That the latter leaves p^μ invariant is readily verified by calculating the transformed energy or momentum for a boost $\bar{u} \perp \hat{z}$.

The little group leaves invariant the form $(t)^2 - (x)^2 - (y)^2$ and $k_0{}^2 - k_x{}^2 - k_y{}^2$. Corresponding to the condition which picked out spacelike three-planes perpendicular to the t axis for ordinary particles--as the spaces in which F_0 displayed its symmetry--is the following condition for tachyons. In the standard frame in which $E_\gamma = 0$ the world line of the tachyon is parallel to the z axis. We define F_0 by first studying its values on the three-planes perpendicular to the z axis, planes which are left invariant under the action of the little group. The 0 in F_0 now refers to the 0 energy in the standard frame. In general, F_0 may be a function of z.

The symmetry of the little group is defined only in the three surface perpendicular to $\hat{z}$; i.e. it consists of rotations about $\hat{z}$ and boosts in the x-y plane.

SO(2,1) symmetry of F_o only imposes the form:

$$F_o(x,y,z,t) = f(t^2-x^2-y^2;z) \text{ or } \tilde{F}_o(k^\mu) = \tilde{f}(k_o^2-k_x^2-k_y^2,k_z)$$

just as O(3) imposed the form:

$$F_o(x,y,z,t) = f(x^2+y^2+z^2,t) \text{ or } \tilde{F}_o(k^\mu) = \tilde{f}(\vec{k}^2,k_o)$$

We take it as given that the form factor of a free tachyon field is an invariant function of the tachyon parameters. The conclusion that the form factor for tachyons is an invariant function only of $t = k^2$ now follows from the observation that F_o picks out no preferred direction in the three-plane perpendicular to $\hat{z}$. The $\hat{z}$ direction itself (the direction of the tachyon velocity) doesn't provide an additional four-vector since it is just proportional to p_T^μ.

Therefore, for Cherenkov radiation for which $t = m_\pi^2$ the form factor is a constant.

For the Fourier transform of the c-number charge distribution $\rho(k^\mu)$, the constancy is seen in another way. In general since $(k^\mu)^2 = (k'^\mu)^2$,

$$\overset{o}{k}{}_o^2 - \overset{o}{k}{}_x^2 - \overset{o}{k}{}_y^2 = k_o^2 - \vec{k}^2 + k_z^2$$

where the $\overset{o}{k}$'s refer to the standard inertial frame. In this frame the tachyon velocity is infinite. If the k^μ refer to a Cherenkov radiated pi particle then we have from Chapter IV, equation (5), for the c number nonrecoiling case:

$$k_o = k_z v_T$$

Hence

$$k_z = k_0/v_T \xrightarrow[v_T\to\infty]{} \overset{\circ}{k}_z = 0$$

and

$$\overset{\circ}{k}{}_0^2 - \overset{\circ}{k}{}_x^{\,2} - \overset{\circ}{k}{}_y^{\,2} = (k^\mu)^2$$

That is, the pi particle is emitted perpendicular to the z axis. Therefore

$$\rho(k^\mu) = \rho_0\left(\overset{\circ}{k}{}_0^2 - \overset{\circ}{k}{}_x^2 - \overset{\circ}{k}{}_y^2, \overset{\circ}{k}_z\right) = \rho_0\left((k^\mu)^2, 0\right) = \rho_0\left(m_\pi^2, 0\right)$$

So we see that both the tachyon charge density $\rho(\bar{k})$ or form factor $F_v(k^\mu)$ are constant for the case of Cherenkov radiation where $t = k^2 = m_\pi^2$, $\overset{\circ}{k}_z = 0$. This suggests the following possibilities;

1) F_0 (or ρ) evaluated at m_π^2 is finite. Therefore F_0 (or ρ) provides no cut off to the radiation leading to divergent Cherenkov energy loss. This implies either that tachyons cannot exist or that they cannot interact in this way.

2) F_0 is zero when evaluated at m_π^2 but finite at some other values of $t = k^2$. Hence there is no Cherenkov radiation at all but the tachyon may interact with other particles through exchange of virtual π's.

3) The analysis of the form factor with noncompact little group SO(2,1) has presented us with another reason to believe there is no sense in considering a free tachyon field even as a first approximation.

To proceed further we take the third point of view and argue that the symmetry of the little group is broken by the interactions and in particular, by the finite world line of the tachyon. According to the arguments based on the little group we found:

$$F_o(x,y,z,t) = F_o(t^2-x^2-y^2,z)$$

$$\text{and} \quad F_o(k^\mu) = F_o(k_o^2-k_x^2-k_y^2,k_z)$$

We assume (analogously with the time dependence of the ordinary particle created and destroyed at t_1 and t_2) that the tachyon form factor does not take on any additional dependence on z in between z_1 and z_2, the end points of its world line. It seems reasonable to assume that the source and sink of the tachyon are at least of finite and probably of microscopic extent in the x-y plane and their motion defines an average inertial frame in the x-y plane. Hence, the symmetry due to the freedom of Lorentz transformations in the x-y plane is lost by the events of creation and annihilation. We assume that the rotational symmetry about the z axis is retained. Hence the form factor (and the charge distribution) in the standard frame is of the form:

$$F\,(x^2+y^2,t)\,\theta\,(z_1,z_2) \tag{4}$$

$$\text{where} \quad \theta(z_1,z_2) = [\theta(z-z_1) - \theta(z-z_2)]$$

Because of the presence of a finite source and sink we have thus lost the noncompactness. To obtain the form factor in any other frame in which the tachyon has velocity v_T we use the scalar property of F_o.

$$F_v(x^\mu) = F_o(\Lambda^{-1}x^\mu) \tag{5}$$

To make a Lorentz transformation in the z direction ($\hat{z} \,//\, \hat{v}_T$) from velocity v_T to infinite tachyon velocity and 0 energy, use:

$$v' = \frac{v-u}{1-uv} = \infty$$

or

$$E_\tau' = (E_\tau - \bar{p}\cdot\bar{u})\gamma_u = E_\tau(1 - v_\tau u)\gamma_u = 0$$

Therefore

$$u = \frac{1}{v_\tau} \qquad \text{and} \qquad \gamma_u = \frac{1}{\sqrt{1-(1/v_\tau)^2}} = \frac{v_\tau}{\sqrt{v_\tau^2 - 1}}$$

and this gives us the Lorentz boost to the standard frame:

$$\mathring{t} = t' = (t - uz)\gamma_u = \left(t - \tfrac{1}{v}z\right)\frac{v_\tau}{\sqrt{v^2-1}} = -(z - v_\tau t)|\gamma_v| \qquad (6)$$

Hence, using (4), (5) and (6):

$$F_v(x^\mu) = F_0\left(x^2+y^2, -(z-vt)|\gamma_v|\right)\Theta(t_1, t_2)$$

we have assumed that the transform of the Θ function in z can be replaced by a Θ function in (t). This implies a certain localizability of F (following from that of the source and sink). The term $(z-v_\tau t)$ does show the dependence one would expect for an object traveling along the z axis with velocity v.

We now see that the noncompact parts of the tachyon form factor are no longer present. They have been avoided beaause of the necessity of having at the ends of the tachyon worldline a tachyon source and sink of finite spatial extent which are not invariant under boosts in the x-y plane.

Appendix D

Beta Decay
Inner Bremsstrahlung Type Radiation
Massive Field Case

In Chapter IV we considered the situation in which a t tachyon is created at a time -T/2 and then destroyed at T/2, displaced by vT along the z direction. No account was taken of the existence of the charge on the source and sink before and after the tachyon world line.

More realistic perhaps, is the situation where the charge exists at rest before the tachyon is created and again displaced by vT after the tachyon is absorbed: i.e.

$$\rho(\bar{r},t) = \begin{cases} \rho(r)e^{\alpha t} \; ; & t < -T/2 \quad (1) \\ \rho(x,y,(z-vt)/|\gamma|) \; ; & -\frac{T}{2} < t < +T/2 \quad (2) \\ \rho(x,y,z-vT)e^{-\alpha t} \; ; & t > T/2 \quad (3) \end{cases}$$

The charge is adiabatically switched on and off at $\pm\infty$ to handle convergence problems in the integrals.

We then have:

$$\phi^{out} = \phi^{out}_{(1)} + \phi^{out}_{(2)} + \phi^{out}_{(3)}$$

$$\phi^{out}_{(1)}(\bar{r},t) = \iint_{-\infty}^{-T/2} \frac{dt'}{(2\pi)^3}\frac{d^3k}{\omega_k} e^{i\bar{k}\cdot(\bar{r}-\bar{r}')} \sin\omega_k(t-t')e^{\alpha t'}\rho(\bar{r}') \qquad (9)$$

$$= \int \frac{d^3k}{(2\pi)^3}\frac{e^{i\bar{k}\cdot\bar{r}}}{(\omega_k)^2}\cos\omega_k(t+T/2)\,\rho(\bar{k}) \qquad (10)$$

$$\phi^{out}_{(2)} = \text{equation IV-25}$$

$$\phi^{out}_{(3)}(\bar{r},t) = \iint_{T/2}^{\infty} \frac{d^3k}{(2\pi)^3}\frac{e^{i\bar{k}\cdot\bar{r}}}{\omega_k}\rho(k)e^{-ik_z vT}\sin\omega_k(t-t')\,e^{-\alpha t'}dt' \qquad (11)$$

$$= -\int \frac{d^3k}{(2\pi)^3}\frac{e^{i\bar{k}\cdot\bar{r}}}{\omega_k^2}\rho(k)e^{-ik_z vT}\cos\omega_k(T/2-t) \qquad (12)$$

Hence, combining $\phi_{(1)}, \phi_{(2)}, \phi_{(3)}$ and picking out the coefficient of $\frac{i}{(2\pi)^{3/2}} \frac{e^{-i\omega_k t + i\vec{k}\cdot\vec{r}}}{(2\omega_k)^{1/2}}$ as before:

$$a_k^{out} = \frac{i\sqrt{2}\sqrt{\frac{v^2}{c^2}-1}}{(2\pi)^{3/2}\sqrt{\omega_k}} \rho\left(k_\perp, \frac{k_z}{|\gamma|}\right) \frac{\sin(\omega_k - k_z v)T/2}{(\omega_k - k_z v)} + \frac{\rho(k)\, e^{-i\omega_k T/2}}{\sqrt{2}(\omega_k)^{3/2}(2\pi)^{3/2}} - \frac{\rho(\vec{k})\, e^{i\omega_k T/2 - i k_z v T}}{\sqrt{2}(\omega_k)^{3/2}(2\pi)^{3/2}} \tag{13}$$

Combining the last two terms we obtain: (multiply first term by $e^{+ik_z vT/2 - ik_z vT/2}$)

$$a_k^{out} = \underbrace{\frac{i\sqrt{2}\sqrt{\frac{v^2}{c^2}-1}}{(2\pi)^{3/2}\sqrt{\omega_k}} \rho\left(k_\perp, \frac{k_z}{|\gamma|}\right) \frac{\sin(\omega_k - k_z v)T/2}{(\omega_k - k_z v)}}_{a^{\check{c}}} - \underbrace{\frac{i\sqrt{2}\,\rho(k)\, e^{-ik_z v\frac{T}{2}} \sin(\omega_k - k_z v)T/2}{(2\pi\omega_k)^{3/2}}}_{a'} \tag{14}$$

Hence:

$$a_k^{out} = i\left(\frac{a^{\check{c}}}{i} - \frac{a'}{i}\cos k_z vT/2\right) - \sin k_z vT/2 \left(\frac{a'}{i}\right) \tag{15}$$

Note that if $v < c$ there is no pole and no Cherenkov radiation. The second term never has a pole if $m_\pi \neq 0$. The second term is the amplitude associated with destroying a charge at $Z=0$, $t = -T/2$, and creating it again at $Z = vT$, $t = T/2$.

From (15) we see that (assume $\rho(-\vec{r}) = \rho(\vec{r})$ so $\rho(\vec{k})$ is real)

$$a_k^+ a_k = |a^{\check{c}}|^2 + |a'|^2 - 2\underbrace{\left(\frac{a^{\check{c}}}{i}\right)\left(\frac{a'}{i}\right)}\cos k_z vT/2 \tag{16}$$

$$-\frac{4\sqrt{\frac{v^2}{c^2}-1}}{(2\pi)^3\omega_k^2}\left[\rho\left(k_\perp \frac{k_z}{|\gamma|}\right)\rho(k)\right]\frac{\sin^2(\omega_k - k_z v)T/2\, \cos k_z v\frac{T}{2}}{(\omega_k - k_z v)}$$

Hence, we see a portion which grows in tim (T), i.e. $|a^{\check{c}}|^2$, which gives the Cherenkov radiation. The remaining part is associated with creating and then destroying the tachyon while also destroying and creating the charge at rest. This is analogous to radiation emitted during beta decay ([62] Jackson) when an electron is created already travelling at an appreciable velocity.

Appendix E

Further Analysis of Generalized Cherenkov Radiation From a C Number Source

We calculate the interpolating field rather than the "out" field in order to see both the radiated and "virtual" particles associated with a superluminal source. $\phi \equiv \phi^{\text{interpolating}}$

$$\phi(\bar{r},t) = \phi^{in}(\bar{r},t) + g\int dt' d^3\bar{r}'\, \Delta^{ret}(\bar{r}-\bar{r}', t-t')\rho(\bar{r}',t') \qquad (1)$$

where ([62] Henley and Thirring):

$$\Delta^{ret}(\bar{r}-\bar{r}', t-t') = \frac{1}{(2\pi)^3}\int d^3\bar{k}\,\frac{e^{i\bar{k}\cdot(\bar{r}-\bar{r}')}}{\omega_k}\sin\omega_k(t-t')\Theta(t-t') \qquad (2)$$

Now, using the result for ρ in equation (IV-21) and switching ρ on adiabatically ($e^{+\alpha t}$)

$$\phi(\bar{r},t) = \phi^{in}(\bar{r},t) + \int\frac{dt' d^3\bar{r}'}{(2\pi)^3}\int_{-\infty}^{t} d^3\bar{k}\,\frac{e^{i\bar{k}\cdot(\bar{r}-\bar{r}')+\alpha t}}{\omega_k}\sin\omega_k(t-t')\rho(x',y',(z-vt)|\gamma|)\,g \qquad (3)$$

Performing the integration over $\bar{r}'$ as in (IV):

$$\phi(\bar{r},t) = \phi^{in}(\bar{r},t) + g\iint_{-\infty}^{t}\frac{d^3k\,dt'}{(2\pi)^3}\frac{e^{i\bar{k}\cdot\bar{r}}}{\omega_k}\sin\omega_k(t-t')\sqrt{\frac{v^2}{c^2}-1}\,e^{-ik_z vt'+\alpha t'}\rho\left(k_x,k_y,\frac{k_z}{|\gamma|}\right) \qquad (4)$$

The integration over t' is

$$\int_{-\infty}^{t}\frac{dt'}{2i}\left[e^{i\omega_k(t-t')-ik_z vt'+\alpha t'} - e^{-i\omega_k(t-t')-ik_z vt'+\alpha t'}\right]$$

$$= \frac{e^{-ik_z vt}}{2}\left[\frac{1}{\omega_k+k_z v+i\alpha} + \frac{1}{\omega_k-k_z v-i\alpha}\right] \qquad (5)$$

Following Henley and Thirring, we write the result in a form which allows us to pick out a_k^+ and a_k, except that here the time dependence is not that of a free field so we seek $a_k(t)$. Make thetchange of variable $\bar{k} \to -\bar{k}$ in the first factor of (5) and in the other terms which multiply it in equation (4). Also assume $\rho(\bar{r})$ is real so that $\rho(-k) = \rho^*(k)$. Then, comparing the two terms (plus ϕ^{in}) with the standard form:

$$\phi(\bar{r},t) = \frac{1}{(2\pi)^{3/2}} \int d^3k \, \frac{e^{i\bar{k}\cdot\bar{r}} a_k(t) + e^{-i\bar{k}\cdot\bar{r}} a_k^+(t)}{(2\omega_k)^{1/2}} \tag{6}$$

we obtain:

$$a_k(t) = a_k^{in} e^{-i\omega_k t} + e^{-ik_z vt} \frac{g\rho\left(k_x, k_y, \frac{k_z}{|\gamma|}\right)\sqrt{\frac{v^2}{c^2}-1}}{\left[(2\pi)^3 2\omega_k\right]^{1/2} (\omega_k - k_z v - i\alpha)} \tag{7}$$

$$a_k^+(t) = a_k^{+in} e^{+i\omega_k t} + (\uparrow)^* \tag{7a}$$

To seperate the "virtual" and created particles, the latter having the free field time dependence, use the relation:

$$\frac{1}{x - x_0 - i\alpha} = P\frac{1}{x - x_0} + i\pi\delta(x - x_0) \tag{8}$$

therefore from (7) and (8)

$$a_k(t) - a_k^{in}(t) = P\frac{1}{(\omega_k - k_z v)} e^{-ik_z vt} \frac{g\rho\left(k_x, k_y, \frac{k_z}{|\gamma|}\right)\sqrt{\frac{v^2}{c^2}-1}}{\left[(2\pi)^3 2\omega_k\right]^{1/2}} + \swarrow \tag{9}$$

$$+ i\pi\delta(\omega_k - k_z v) e^{-i\omega_k t} \frac{g\rho\left(k_x, k_y, \frac{k_z}{|\gamma|}\right)\sqrt{\frac{v^2}{c^2}-1}}{\left[(2\pi)^3 2\omega_k\right]^{1/2}}$$

That the second term in equation (9) gives the real, created particles is seen by the time dependence or by the presence of the delta function, which we saw in (IV-30) will give a linear growth in time of the number, when it is squared.

However, there is another interesting aspect of the relation between real and "virtual" particles which equation (9) demonstrates. The second term as a factor of i which the first term lacks. We would like the total number of particles to equal the number of "virtuals" plus the number of real particles so that it makes sense to distinguish between them. But we also want the amplitudes to be additive. So if we write:

$$a_{TOTAL} = \sqrt{n_{virtual}}\, e^{i\phi_v} + \sqrt{n_{real}}\, e^{i\phi_r}$$

and require

$$a_T^+ a_T = N_{TOTAL} = n_{virtual} + n_{real}$$

then we find:

$$\phi_r = \phi_v \pm \pi/2$$

This serves as an additional justification for considering <u>only</u> the second term in equation (9) as representing the created particles, since the first term differs by $\pi/2$ in phase. The first term is the reactive part, the second term the "resonance" part of the field response to the superluminal object.

Using the relation for the square of a delta function and equation (9) above, we arrive at equation (IV-31) for dn_k/dt.

The total number of "virtual" particles is, from equation (7) and (7a):

$$\int d^3k\, n_k^{virt.} = \int d^3k\, \frac{g^2 \left|\rho\left(k_x, k_y, \frac{k_z}{|\gamma|}\right)\right|^2}{(2\pi)^3\, 2\omega_k} \frac{\left(\frac{v^2}{c^2}-1\right)\left(\omega_k - k_z v\right)^2}{\left[\left(\omega_k - k_z v\right)^2 + \alpha^2\right]^2} \qquad (10)$$

Where, in equation (10), we retain the factors of α^2 in order to give a means of dealing with the square of a Cauchy principal value. Now assume cylindrical symmetry, and change the variables from $k_\perp$, k_z, to ω_k, k_z which yields: ($\frac{\partial \omega}{\partial k_\perp} = \frac{k_\perp}{\omega_k}$)

$$\int_0^{2\pi} d^3k = 2\pi k_\perp dk_\perp dk_z = 2\pi\, \omega_k d\omega_k dk_z$$

This is rather obvious but can also be verified by computing the Jacobian.

$$\therefore \quad N^{virtual} = \int \frac{d\omega_k dk_z}{(2\pi)^2} \frac{|\rho()|^2}{2} \frac{\left(\frac{v^2}{c^2}-1\right)(\omega_k - k_z v)^2}{\left[(\omega_k - k_z v)^2 + \alpha^2\right]^2} \tag{11}$$

Since $\omega_k^2 = k_\perp^2 + k_z^2 + m_\pi^2$, $k_\perp^2 \geq 0 \Rightarrow \omega_k \geq m_\pi$. Note that if we first set $\alpha = 0$ the number of virtual particles for $v > c$ diverges. If however, we first assume that ρ is approximately constant up to an ω_{max} and $k_{z\,max}$ and essentially zero beyond, we can investigate further the dependence on α . Assume α is small compared with $k_{z\,max}$. Then a little algebra transforms the integral over k_z:

$$\int_{-k_{z\,MAX}}^{k_{z\,MAX}} dk_z \frac{|\rho|^2}{2} \frac{(\omega_k - k_z v)^2}{\left[(\omega_k - k_z v)^2 + \alpha^2\right]^2} = \frac{1}{\alpha}\int_{-k_{z\,MAX}}^{k_{z\,MAX}} \frac{dk_z}{\alpha} \frac{|\rho|^2}{2} \frac{\left(\frac{\omega_k}{\alpha} - k_z v/\alpha\right)^2}{\left[\left(\frac{\omega_k}{\alpha} - k_z v/\alpha\right)^2 + 1\right]^2} \tag{12}$$

$$= \frac{1}{\alpha}\int_{-\frac{k_{z\,MAX}}{\alpha}}^{\frac{k_{z\,MAX}}{\alpha}} dx \frac{(x_0 - xv)}{\left[(x_0 - xv)^2 + 1\right]^2}$$

The limits on the integral may be taken to be approximately $\pm\infty$ by our assumption about α . The integral over ω_k in equation (11) now simplifies since the ω_k factors drop out after the integration from $-\infty \to +\infty$ over $\frac{k_z}{\alpha}$. We have then approximately:

$$\mathcal{N}^{\text{virtual}} \simeq \underline{\underline{\frac{1}{\alpha}}}\, \frac{g^2}{2} \left(\omega_{max} - m_\pi\right) A \simeq g^2 \left(\omega_{max} - m_\pi\right) \frac{A}{2}\, \underline{\underline{T}} \qquad (13)$$

The constant A contains the (finite) value of the integral over k_z/α and the factors of v, etc. By the term $1/\alpha$ we see that the number of "virtuals" for a Cherenkov radiating particle ($v > c$) grows linearly in time, as does the number of real particles as found in (IV-31). Note that this is true for $m_\pi \neq 0$ in general, assuming ρ contains the requisite Fourier components.

In order to interpret this, look at equation (11) in the case v = 0, taking care to restore factors of $\frac{v^2}{c^2} - 1 \rightarrow 1 - \frac{v^2}{c^2}$. In the case $m_\pi \neq 0$ there is no singularity for ~~for~~ $\alpha = 0$ since $\omega_k \geqslant m_\pi$. However, for $m_\pi = 0$ the singularity that arises is the well known infrared catastrophe appearing here in the switching on of the charge rather than in bremsstrahlung. From equation (11) one sees that the divergence is of the form dk/k as usual.

When we retain the factor of α the growth of $\mathcal{N}^{\text{virtual}}$ can be seen in k space to be owing to the behaviour of the Fourier transform of the time dependent ρ , which approaches resonance ($\omega = 0$) as $t \rightarrow \infty$. In ordinary space the growth of $\mathcal{N}^{\text{virtual}}$ is associated with the retardation of the field, ϕ being zero for $r > ct \approx c/\alpha$ and increasing for $r < ct$ because of the switching on of the charge.

In figure (13) we compare the situation to that of an undamped harmonic oscillator which is driven at various frequencies. The condition $\omega_k = k_z v$ corresponds to driving on resnnance, and for a harmonic oscillator leads to an amplitude growing linearly in time. The other, neighboring frequencies are not on resonance, but are so close that they also give rise to large amplitudes. Also, the closer to resonance the longer it would take to reach the maximum amplitude: in fact, it would be proportional to the inverse of the beat frequency $\frac{1}{\omega_k - k_z v}$. The general solution for an undamped driven Harmonic Oscillator, if the resonance frequency is denoted by ω_k and the driving force has frequency $k_z v$, is:

$$x = C\cos\omega_k t + D\sin\omega_k t + \frac{\frac{F_0}{m_{H.O.}}}{\omega_k^2 - (k_z v)^2}\cos(k_z v t + \beta) \tag{14}$$

where C. D, β relate to initial conditions: and if $\omega_k = k_z v$, i.e. on resonance:

$$x = C\cos\omega_k t + D\sin\omega_k t + \frac{F_0 t}{2 m_{H.O.}\omega_k}\sin(\omega_k t + \beta) \tag{15}$$

Note the $\pi/2$ phase difference in the driven term and compare our previous comments on the essential difference between "reactive" and "resonant" response. Also note in this case that the "catastrophe" exists at frequencies approaching the resonant frequency. (equation (14))

Hence we view the "Cherenkov catastrophe" of a divergent (if $\alpha = 0$) or linearly increasing (if $\alpha \sim \frac{1}{t}$ is small but finite) number of virtual particles as due to the infrared catastrophe "transformed" to superlight velocity. (i.e., if v = 0 the pole

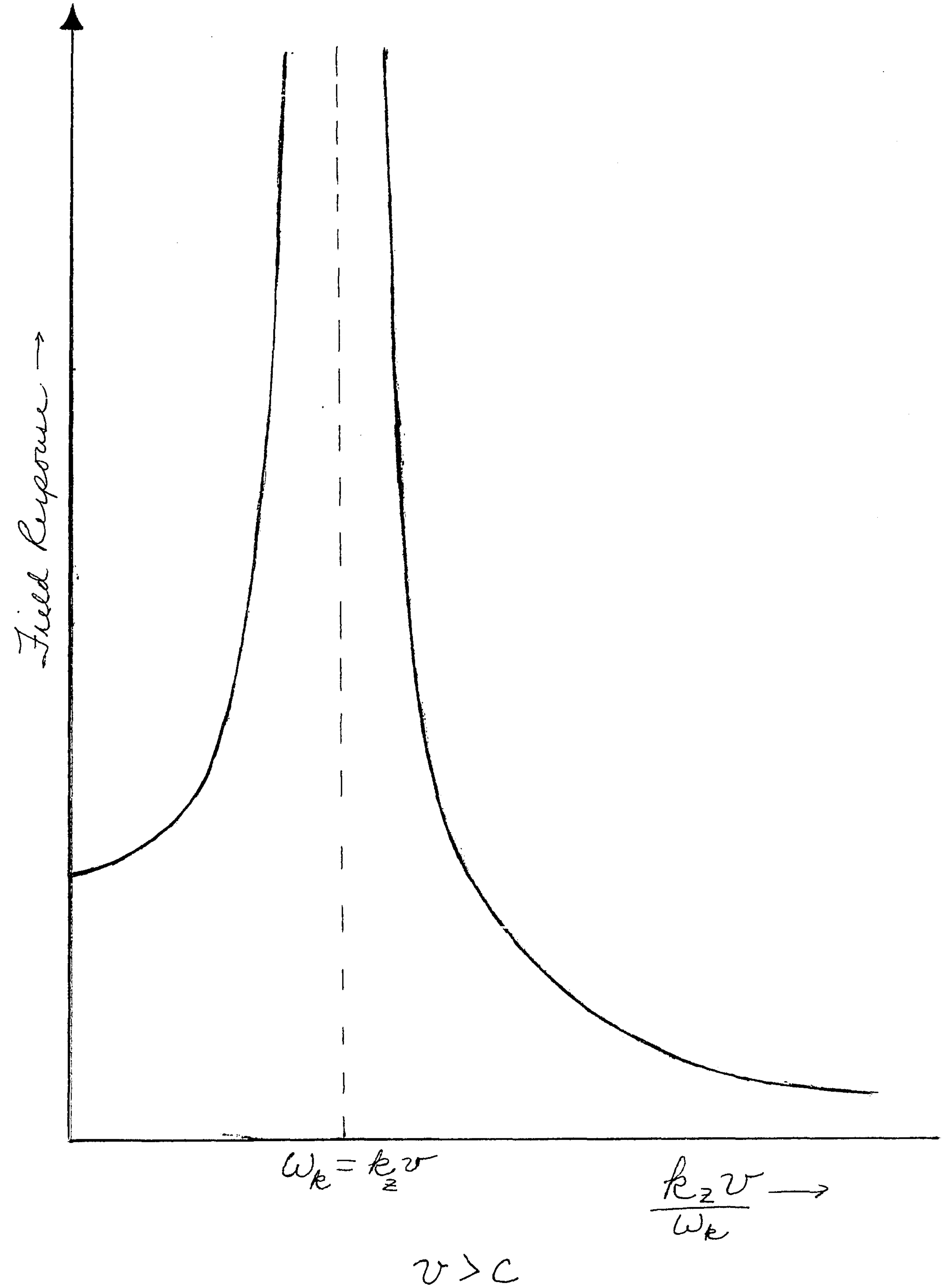


figure 13

is at $\omega = 0$. This transforms for $v > c$ to a pole at $\omega_k = \frac{k}{2} v$, But, whereas there is no infrared catastrophe for $m_\pi \neq 0$ and $v = 0$, we do find it for any m_π if $v > c$. Furthermore, the super-luminal case would not be subject to the usual interpretation of the infrared divergence because the energy of the diverging number of virtual quanta does not go to zero.

In ordinary space these diverging "virtual " quanta are apparently associated with the parts of the field within the Cherenkov cone which are very near the shock cone.

Although we have distinguished between real and "virtual" quanta here, the fact that they are arbitrarily close to the mass shell indicates that there is no clear cut difference between them other than the phase factor that we noted.

We will now show that, depending on how the source is switched on and off, they may or may not contribute to the Cherenkov radiation actually detectable. For a realistic switching we will see that they do in fact contribute: i.e. for sudden switching we show that the "virtuals" are effectively real.

To further investigate the details of generalized Cherenkov radiation, we will compare a number of different ways of switching on the charge for different tachyon motions. We will calculate the asymptotic fields generated by these and refer to the above analysis of the interpolating field for comparison.

First, we adiabatically switch on the charge at rest until $t = 0$ and then jump to $v > c$ and adiabatically switch it off. Note that we use different switching rates.

$$\rho = \begin{cases} \rho(r) e^{\alpha t} & \text{for } t<0 \\ \rho(x,y,(z-vt)|\gamma|) e^{-\beta t} & \text{for } t>0 \end{cases}$$

$$\therefore \quad \phi^{out} = \phi_0^{in} + \int_{-\infty}^{0} + \int_{0}^{\infty}$$

$$\int_{-\infty}^{0} = \int_{-\infty}^{0} \frac{1}{(2\pi)^3} \int d^3k \, e^{i\bar{k}\cdot(\bar{r}-\bar{r}')} \frac{\sin\omega_k(t-t')}{\omega_k} e^{\alpha t'} \rho(\bar{r}') \, dt' \, d^3\bar{r}'$$

$$= \frac{1}{(2\pi)^3} \int_{-\infty}^{0} \frac{d^3k}{\omega_k} \frac{e^{i\omega_k(t-t')+\alpha t'} - e^{-i\omega_k(t-t')+\alpha t'}}{2i} \rho(\bar{k}) \, dt'$$

$$= \frac{1}{(2\pi)^3} \int d^3k \frac{e^{i\bar{k}\cdot\bar{r}}}{\omega_k 2i} \rho(k) \left[\frac{e^{i\omega_k t}}{\alpha - i\omega_k} - \frac{e^{-i\omega_k t}}{\alpha + i\omega_k} \right]$$

$$= \frac{1}{(2\pi)^3} \int d^3k \frac{e^{i\bar{k}\cdot\bar{r}} \rho(\bar{k})}{\omega_k \, 2i} \left[e^{i\omega_k t} \frac{\alpha + i\omega_k}{\alpha^2+\omega_k^2} - e^{-i\omega_k t} \frac{(\alpha - i\omega_k)}{\alpha^2+\omega_k^2} \right]$$

and

$$\int_0^{\infty} = \int_0^{\infty} \frac{d^3k}{(2\pi)^3} \frac{dt'}{\omega_k} e^{i\bar{k}\cdot(\bar{r}-\bar{r}')} \rho(x',y',(z'-vt')|\gamma|) e^{-\beta t'} \sin\omega_k(t-t') \, d^3\bar{r}'$$

make the change of variable: $z' = \frac{\xi}{|\gamma|} + vt'$, $dz' = \frac{d\xi}{|\gamma|}$ **as in Chapter IV.**

$$\int_0^{\infty} = \int_0^{\infty} \frac{d^3k}{(2\pi)^3} dt' \, e^{i\bar{k}\cdot\bar{r} - ik_z vt' - \beta t'} \left(\sqrt{\frac{v^2}{c^2}-1}\right) \left[\frac{e^{i\omega_k(t-t')} - e^{-i\omega_k(t-t')}}{2i} \right] \rho\left(k_\perp, \frac{k_z}{|\gamma|}\right)$$

$$= -\int \frac{d^3k}{(2\pi)^3} \frac{e^{i\bar{k}\cdot\bar{r}}}{\omega_k 2i} \left[\frac{e^{i\omega_k t}}{-i\omega_k - ik_z v - \beta} - \frac{e^{-i\omega_k t}}{+i\omega_k - ik_z v - \beta} \right] \rho\left(k_\perp, \frac{k_z}{|\gamma|}\right) \sqrt{\frac{v^2}{c^2}-1}$$

$$\therefore \quad a_k^{out} = \frac{\rho(k)(\omega_k + i\alpha)}{(2\pi)^{3/2}\sqrt{2\omega_k}(\omega_k^2+\alpha^2)} - \rho\frac{\left(k_\perp, \frac{k_z}{|\gamma|}\right)\sqrt{\frac{v^2}{c^2}-1}}{(2\pi)^{3/2}\sqrt{2\omega_k}} \frac{[(\omega_k - k_z v) + i\beta]}{[(\omega_k - k_z v)^2 + \beta^2]} \qquad (16)$$

In order for the second term to give the same Cherenkov rate as in (IV) if we take $\frac{1}{\beta} = T$ then the $\left[P\left(\frac{1}{x}\right)\right]^2$ term must contribute an amount equal to the $[\delta(x)]^2$ term in (16).

Compare equation (7a) where we adiabatically switched on the tachyon charge from $-\infty$ and calculated ϕ (interpolating field) We see that except for a sign change, the second term of (16) is

the same as the term in 7a

In 7a the two parts of a, i.e. the $\delta(x)$ and $P\left(\frac{1}{x}\right)$ parts don't necessarily have the same time dependence. Here we see that if we switched off the source at t = 0, the "virtuals," i.e. $P\left(\frac{1}{x}\right)$ term, would take on the free field time dependence $e^{-i\omega_k t}$ and appear as created particles.

To check this we now calculate ϕ^{out} for a v > c particle adiabatically switched on from $-\infty$ and then destroyed at 0. Using results from above for ρ etc.

$$\phi^{out} = \int_{-\infty}^{0} \frac{d^3k}{(2\pi)^3} \frac{e^{i\vec{k}\cdot\vec{r}}}{\omega_k} \sqrt{\frac{v^2}{c^2}-1}\; \rho\left(k_\perp, \frac{k_z}{|\gamma|}\right) e^{\alpha t' - i k_z v t'} \sin \omega_k (t-t')\, dt'$$

$$= \int \frac{d^3k}{(2\pi)^3} \frac{e^{i\vec{k}\cdot\vec{r}}}{\omega_k} \sqrt{\frac{v^2}{c^2}-1}\; \frac{\rho\left(k_\perp, \frac{k_z}{|\gamma|}\right)}{2i} \left[\frac{e^{i\omega_k t}}{\alpha - i k_z v - i\omega_k} - \frac{e^{-i\omega_k t}}{\alpha - i k_z v + i\omega_k}\right]$$

Hence:

$$a_k^{out} = \frac{\sqrt{\frac{v^2}{c^2}-1}\; \rho\left(k_\perp, \frac{k_z}{|\gamma|}\right)}{(2\pi)^{3/2}\sqrt{2\omega_k}} \frac{\left[(\omega_k - k_z v) + i\alpha\right]}{(\omega_k - k_z v)^2 + \alpha^2}$$

and:

$$(a^+a)_k^{out} = \left(\frac{v^2}{c^2}-1\right) \frac{\left|\rho\left(k_\perp, \frac{k_z}{|\gamma|}\right)\right|^2}{(2\pi)^3\, 2\omega_k} \frac{(\omega_k - k_z v)^2 + \alpha^2}{\left[(\omega_k - k_z v)^2 + \alpha^2\right]^2}$$

This is seen to be equal to the sum of what we called the real and the "virtual" particles (Compare (10)).

Hence, we conclude, since the "virtuals" are so close to the mass shell and also grow with time, that they contribute to the measured Cherenkov radiation as well. This is an interesting point since a) the meaning of real and virtual is seen not to be clearly distinct here, and since b) if we adiabatically also switched the charge on and then off the reactive part would have cancelled out if $\alpha = \beta$; i.e. if we had equal rates of switching. To see this, note:

$$\int_{-\infty}^{\infty} e^{i(\omega_k - k_z v)t' - \alpha|t'|}\, dt' = \frac{1}{i(\omega_k - k_z v) + \alpha} - \frac{1}{i(\omega_k - k_z v) - \alpha}$$

$$= + \frac{2\alpha}{(\omega_k - k_z v)^2 + \alpha^2}$$

This corresponds to the "resonant" term in the previous equation. Hence, the "reactive" part which gave rise to the Principal Value term previously (df. equation (10)), is seen to be damped out if the rate of switching on equals the rate of switching off. Note that if the rates were different, we would not get this cancellation, (Compare H.O. analog). The factor of 2α is needed here since this time of radiation is twice as long.

Appendix F

Massive Cherenkov Radiation from a Thin Spherical Shell

For a thin spherical shell of radius r_0 the generalized charge density is:

$$\rho(\bar{r}) = g\,\frac{\delta(r-r_0)}{4\pi r_0^2} \tag{1}$$

Hence the Fourier transform is:

$$\begin{aligned}\rho(\bar{k}) &= \int d^3\bar{r}\, e^{i\bar{k}\cdot\bar{r}}\rho(\bar{r}) = \int_0^{\pi} 2\pi \sin\theta\, d\theta\, e^{ikr\cos\theta}\, g\,\frac{\delta(r-r_0)}{4\pi r_0^2}\, r^2\, dr \\ &= g\,\frac{\sin kr_0}{kr_0}\end{aligned} \tag{2}$$

Substituting this in eqaation IV. (36) we have the integral:

$$g\int_{m_\pi}^{\infty} k_z'\, dk_z'\, \frac{\sin^2 kr_0}{(kr_0)^2}$$

where:

$$k = \sqrt{k_\perp'^2 + k_z'^2} = \sqrt{2k_z'^2 - m_\pi^2}$$

Although the value of the integral is affected for finite k_z' by the presence of nonzero m_π, it is seen that for large k_z' the integral is logarithmically divergent. This agrees with Sommerfeld's divergent energy loss for E-M radiation.

$$\frac{dE}{dt} \rightarrow \infty$$

Appendix G

Electromagnetic Cherenkov Radiation for Hyperbolic Motion

The angle at which Cherenkov radiation is emitted has been derived for constant velocity of the source. In a medium it has been shown that the quantized electromagnetic field gives rise to a quantum correction to the angle which is a function of frequency. We pointed out however, that in a vacuum where n = 1 and $v > c$ this correction is not present. This is due, essentially, to the fact that $v_{\gamma} = 1$ and hence, $k^{\mu} = (\omega, \frac{\omega}{c})$. But, there is still recoil since $\hbar \neq 0$. We have shown that due to this recoil the tachyon undergoes hyperbolic motion and accelerates to ∞ velocity.

In order to see what effect this acceleration has on the direction of the Cherenkov radiation, we seek the form of the wave front. For constant velocity the construction is simple and turns out to be a straight line ([58] Jelley). The Huygens construction ([62] Courant & Hilbert) yields the angle $\cos\Theta = c/v_{\gamma}$.

For hyperbolic motion the situation is complicated by the acceleration. In order to cope with this we make use of ([61] Widder) Leibnitz's method for calculating the envelope of a family of curves. If a member of the family is given by the equation (with parameter c).

$$\phi(z, y, c) = 0 \tag{1}$$

Then the envelope is found by eliminating the parameter c between equation (1) and:

$$\frac{\partial\phi}{\partial c} = 0 \tag{2}$$

In the present case the curves are cirules centered on the retarded position of the tachyon on the z axis. The radii correspond to the distance light would have traveled in the elapsed time. See figure 14.

We now seek the shape of the envelope at the time at which the particle is at z = 0, y = 0, x = 0 and has velocity equal to infinity. In Chapter III we derived the equation for (tachyonic) hyperbolic motion.

$$z = -\sqrt{t^2 - g^{-2}} \quad \text{and} \quad \frac{dz}{dt} = -\frac{t}{\sqrt{t^2 - g^{-2}}} = \frac{t}{z} \tag{3}$$

In order to indicate the method, we assume that the motion is rectilinear. One can also calculate the envelope for a time at which the tachyon energy is very high. We are looking at a two dimensional crossection (z,y plane) with the particle approaching the origin from the negative z axis. From equation (3) we see that the tachyon reaches the origin and infinite velocity at $t = -g^{-1}$, which is the time at which we seek the envelope. The equation for the family of circles centered about $z = -\sqrt{t^2 - g^{-2}}$, y = 0 and with radius equal to $c(t+g^{-1})$ (so at $t = -g^{-1}$ the radius = 0) is

$$\left(z + \sqrt{t^2 - g^{-2}}\right)^2 + y^2 = c^2\left(t + g^{-1}\right)^2 \tag{4}$$

The parameter of this family is t, hence $\frac{\partial \phi}{\partial t} = 0$ from (2) yields

$$\frac{t}{\sqrt{t^2 - g^{-2}}}\left(z + \sqrt{t^2 - g^{-2}}\right) - \left(t + g^{-1}\right) = 0 \tag{5}$$

Eliminating t between these two equations with some algebra (remember z is negative) yields the equation of the envelope: (figure 14)

$$\left(y \pm g^{-1}\right)^2 + z^2 = g^{-2} \tag{6}$$

This is a half circle with radius g^{-1} and center at $(z,y) = (0,\pm g^{-1})$ In three dimensions the two half circles should be rotated about the z axis to sweep out the surface of the wave front. It is seen in equation (6) that the same result holds for positive z as well, and z = 0 is the point at which the two tachyons annihilate. The figure shows that the Cherenkov radiation is focused (in three dimensions) at a ring of radius g^{-1} centered about z = 0 in the x-y plane.

Looking at a position z(t) at an arbitrary time t (see geometric construction in figure (14). We see that (use equation (3)):

$$\cos\theta = \frac{z(t)}{c|t| - g^{-1} + g^{-1}} = \frac{c}{v(t)} \tag{7}$$

Hence, we see that, at least for hyperbolic motion the direction of radiation is given by the usual Cherenkov relation at each instant. From the point of view of the four-momentum conservation derivation in (III), this seems reasonable.

For any other time one could also solve for the envelope (i.e. when $v < \infty$) in the same manner. Also, if z(t) is any other function of time written in a tractable form, equations (1) and (2) may be solvable.

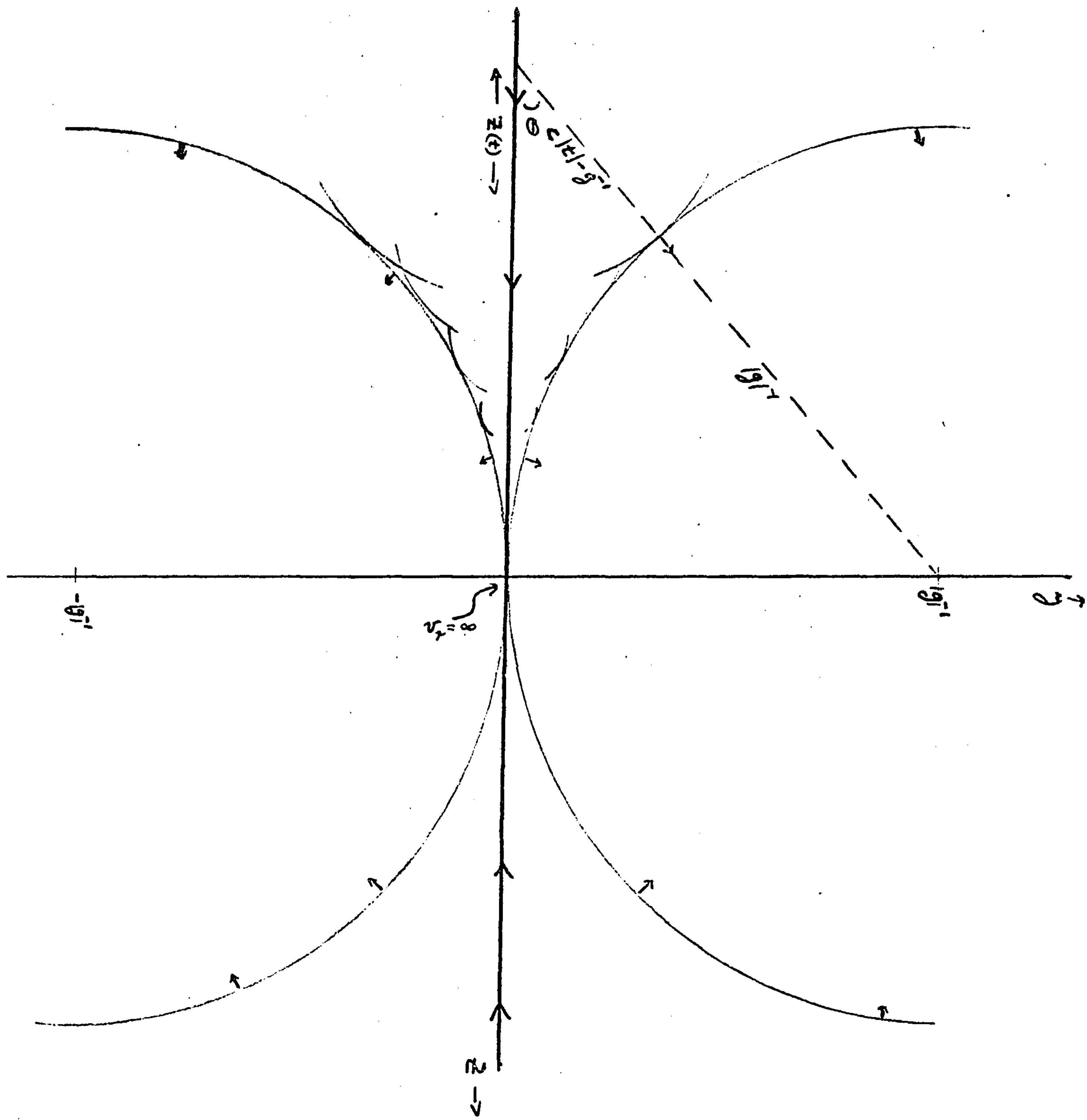
For ordinary Cherenkov radiation in a medium with $c > v > c/n$ undergoing ordinary hyperbolic motion the relevant equations are: (assume slowing down through c/n)

$$z^2 - t^2 = a^{-2}$$
$$\left(z + \sqrt{t^2 + a^{-2}}\right)^2 + y^2 = \left(\frac{c}{n}\right)^2 t^2 \tag{8}$$

When one solves for the envelope of Cherenkov radiation the result is a hyperbola $\frac{-z^2\left(\frac{c}{n}\right)^2}{1-(c/n)^2} + y^2 + a^{-2}\left(\frac{c}{n}\right)^2 = 0$

For Cherenkov radiation of a massive field use the phase velocity for the π field to obtain the wave front of each k component.

Rockower (1)



$$(y \pm |g|^{-1})^2 + z^2 = |g|^{-2}.$$

Cherenkov Radiation from Hyperbolic ($v > c$) motion

figure 14

H

Motion of a Tachyon in a Constant Magnetic or Electric Field

Magnetic Field:

The motion of a tachyon in a constant magnetic or electric field is calculated. The tachyon is treated as a classical particle. Although it is assumed to be acted upon by the usual electric or magnetic force appropriate to a particle of charge e and velocity v_T, we assume for this calculation that there is no Cherenkov radiation.

We first treat the case of a magnetic field H which is along the z axis.

$$\dot{\bar{p}} = \frac{e\bar{v}_T \times \bar{H}}{c} \qquad \text{and} \qquad \bar{p} = \frac{T\bar{v}_T}{c^2}$$

where T, the kinetic energy is constant in a magnetic field H.

$$\therefore \quad \frac{T}{c^2}\frac{dv}{dt} = \frac{e}{c}\bar{v} \times \bar{H}$$

or

$$\dot{v}_x = \omega v_y \;,\quad \dot{v}_y = -\omega v_x \;,\quad \dot{v}_z = 0$$

where:

$$\omega = \frac{ecH}{T}$$

Hence:

$$\frac{d}{dt}(v_x + i v_y) = -i\omega(v_x + i v_y)$$

and solving this:

$$v_x + i v_y = v_{0t}\, e^{-i(\omega t + \alpha)} \qquad \text{or: } v_x = v_{0t}\cos(\omega t + \alpha)$$

in which:

$$v_{0t} \equiv \sqrt{v_{0x}^2 + v_{0y}^2}$$

$$\therefore \quad x = x_0 + r\sin(\omega t + \alpha)\,, \quad y = y_0 + r\cos(\omega t + \alpha)$$

and:

$$r \equiv \frac{v_{0t}}{\omega} = \frac{v_{0t}\,T}{ecH}$$

This relation is seen to be the same as that for an ordinary particle in a magnetic field ([62] Landau and Lifshitz, hereafter denoted L & L). That this should be true is indicated by our equation (9) from Chapter III $\frac{d\bar{v}}{dt} = \frac{c^2\bar{F}}{E} \cdot \left(\bar{\bar{I}} - \frac{\bar{v}\bar{v}}{c^2}\right)$. Note that for a magnetic force the second term will drop out since $\bar{F} \propto \bar{v} \times \bar{H}$.

For an electric field the situation has some new features.

Electric Field:

We assume the electric field $\bar{E}$ is constant and in the x direction. The motion is in a plane which we take to be the x-y plane. The equations of motion for the tachyon are:

$$\dot{p}_x = e\bar{E} \quad , \quad \dot{p}_y = 0$$

Hence:

$$p_x(t) = eEt + p_{0x} \quad , \quad p_y(t) = p_{0y}$$

The tachyon kinetic energy is:

$$T = \sqrt{(p_{0x} + eEt)^2 + p_{0y}^2 - m^2} \tag{1}$$

We see from this that there are a number of different cases, depending on the tachyon parameters e, $\bar{p}_0$. If $|p_{0y}|$ is greater than m_T then T will always be greater than zero and the motion will be similar to that of an ordinary particle; $|p_{0y}| \leq m_T$ leads to new solutions.

The tachyon velocity is, in the x direction:

$$\frac{dx}{dt} = v_x = \frac{p_x c^2}{T} = \frac{p_{0x} + eEt}{\sqrt{(p_{0x} + eEt)^2 + p_{0y}^2 - m^2}}$$

Therefore:

$$x = \int_0^t dt \, \frac{p_{0x} + eEt}{\sqrt{\cdots}}$$

Hence:

$$x = \frac{1}{eE}\sqrt{\left(p_{ox}+eEt\right)^2+p_{oy}^2-m^2} \tag{2}$$

Similarly we find for y:

$$\frac{dy}{dt} = \frac{p_y c^2}{T} = \frac{p_{oy} c^2}{\sqrt{}}$$

and:

$$y = \int_0^t dt\, \frac{p_{oy} c^2}{\sqrt{(p_{ox}+eEt)^2+p_{oy}^2-m^2}} = \frac{1}{eE}\int_{p_{ox}}^{p_{ox}+eEt} \frac{du\, p_{oy} c^2}{\sqrt{u^2+p_{oy}^2-m^2}} \tag{3}$$

The form of the solution to this integral will depend on the relative magnitude of $|p_{oy}|$ and m.

case a) $|p_{oy}| > m$

Set $p_{oy}^2 - m^2 = \phi^2$, then (3) becomes:

$$y - y_0 = \frac{1}{eE}\int \frac{du/\phi\; p_{oy} c^2}{\sqrt{\frac{u^2}{\phi^2}+1}} = \frac{p_{oy}c}{eE}\sinh^{-1}\left(\frac{p_{ox}+eEt}{\sqrt{p_{oy}^2-m^2}}\right) \tag{4}$$

Combining (2) and (4) to eliminate t we obtain:

$$x = \frac{\sqrt{p_{oy}^2-m^2}}{eE}\cosh\left(\frac{(y-y_0)}{p_{oy}c/eE}\right) \tag{5}$$

This is a catenary which is of the same form found for ordinary particles ([62] L & L p. 58)

We now look at the case for which $|p_{oy}| < m$,

case b)

The solution for x as a function of t is still given by equation (2). To solve for y substitute $m^2 - p_{oy}^2 = \psi^2$ in equation (3)

$$y = \frac{1}{eE}\int_{p_{ox}}^{p_{ox}+eEt} \frac{dU/\psi \; p_{oy} c}{\sqrt{(U/\psi)^2 - 1}}$$

Hence

$$y - y_0 = \frac{p_{oy} c}{eE} \cosh^{-1}\left(\frac{p_{ox} + eEt}{\sqrt{m^2 - p_{oy}^2}}\right) \qquad (6)$$

and combining with equation (2) in order to eliminate t:

$$x = \frac{\sqrt{m^2 - p_{oy}^2}}{eE} \sinh\left(\frac{y - y_0}{p_{oy} c/eE}\right) \qquad (7)$$

Now if $p_{oy} = m$, case c), we find from (2)

$$x = \frac{p_{ox}}{eE} + ct$$

and from (3):

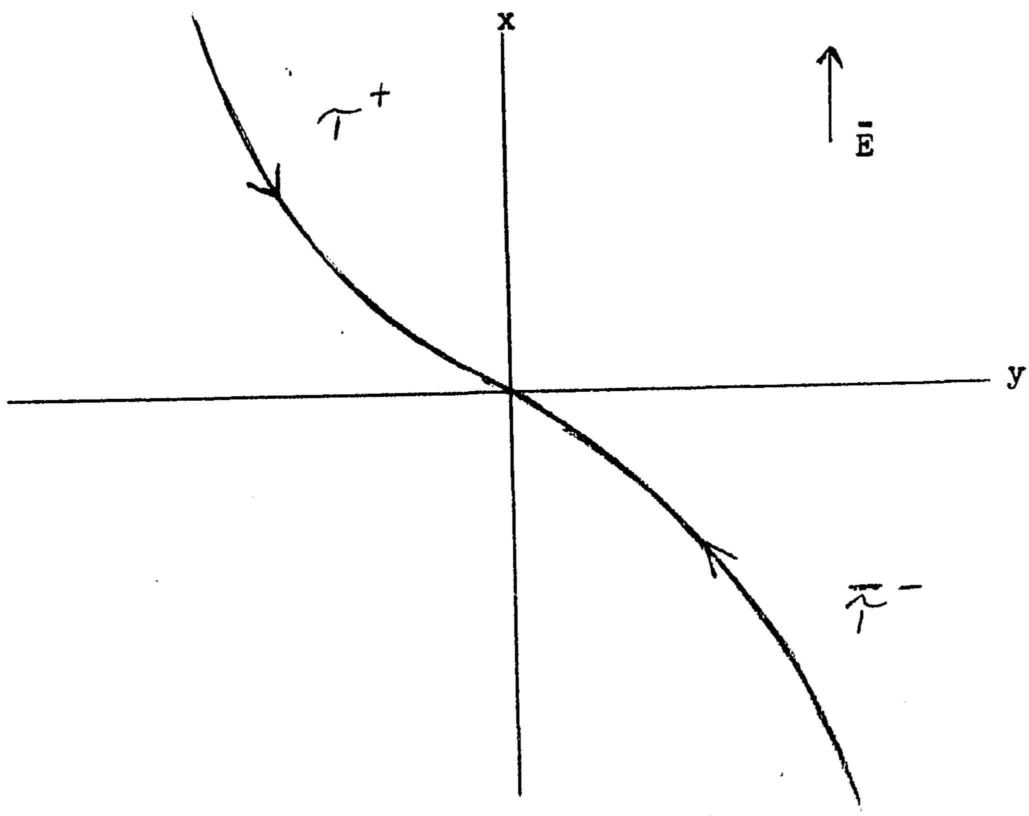
$$y = \frac{p_{oy} c^2}{eE} \ln\left(\frac{p_{ox} + eEt}{p_{ox}}\right)$$

Hence:

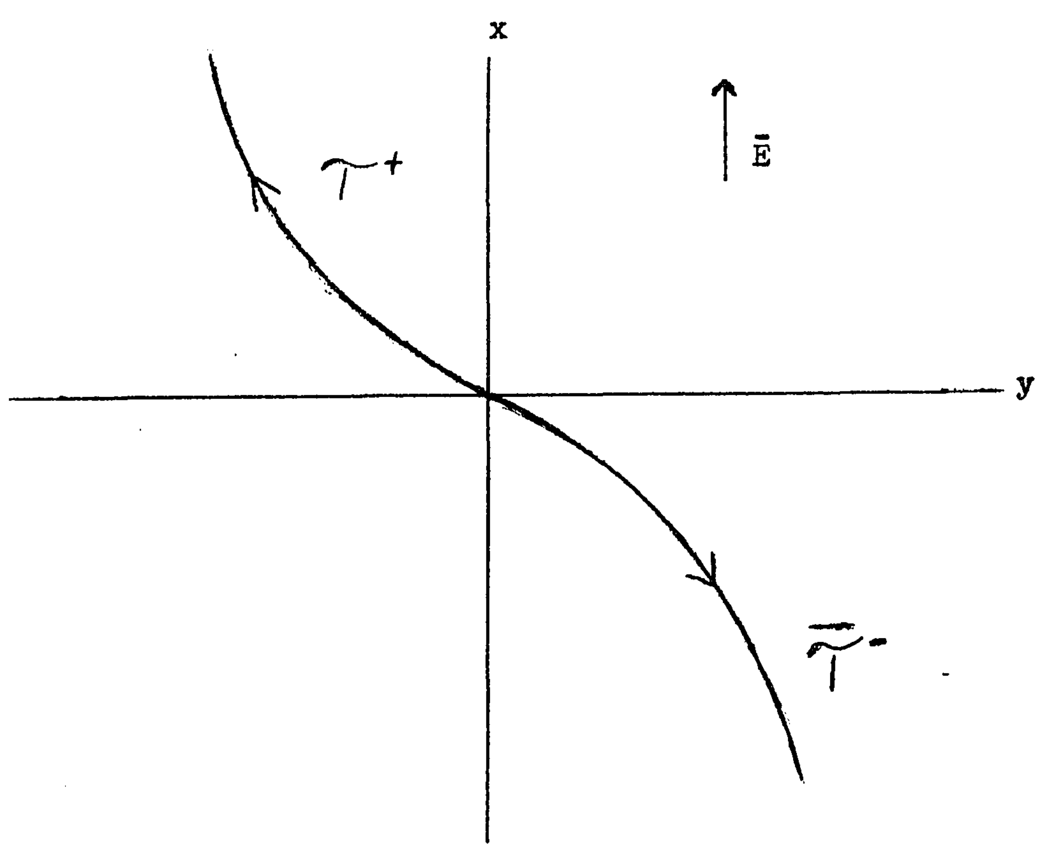
$$x = \frac{p_{ox}}{eE}\left[e^{eEy/p_{oy}c^2} - 1\right]$$

Note in equation (2) that for case a) $(|p_{oy}| > m)$ there exists a real solution for x for all times t. However for case b) $(|p_{oy}| < m)$ there exists no solution after a certain time if p_{ox} and e have opposite signs. This corresponds to the necessity of having an oppositely charged antitachyon annihilate the tachyon when the kinetic energy T becomes zero. See figure (15a). We also see from equation (2) in case b) that there may exist no solution before a certain time if $\dot{\gamma}_{ox}$ and e have the same sign, but that x then goes to plus or minus infinity as $t \to \infty$. See figure 15b. Equation (7) describes both of these situations.

Figure 15b shows that a uniform electric field can create $\tau - \bar{\tau}$ pairs. This is possible because there is no minimum energy required; they both have zero energy. Charge, energy and any other quantum number would be conserved. This instability of the tachyon vacuum in an electric field provides another reason for believing that tachyons cannot exist as free particles.



(a)



(b)

figure 15

Footnotes

1. A number of the early papers on charges with superlight velocity are inclùded in the extensive tachyon bibliography compiled by Dr. Eleanor Maas of Swarthmore College for Dr. Bilaniuk, which they kindly made available to me; included are [04] Sommerfeld, [89] Thomson, [92] Heaviside.

2. We find the asymptotic expansion ([69] Whittaker & Watson) of $\psi_T(\bar{r},t)$ by integrating by parts repeatedly. For convenience we can assume $f(\bar{k})$ to be spherically symmetric.

$$\psi(\bar{r},0) = \int e^{i\bar{k}\cdot\bar{x}} \frac{f(\bar{k})\, d^3k}{\sqrt{k^2-m^2}} = \int e^{ikx\cos\theta} \frac{f(\bar{k})\, 2\pi k^2 d(\cos\theta)\, dk}{\sqrt{k^2-m^2}}$$

In this section only we take $|\bar{k}| \equiv k$.

$$\psi(\bar{r},0) = \int \sin \frac{kr \cdot \pi f(k)\, k^2 dk}{kr\sqrt{k^2-m^2}}$$

$$\doteq -\frac{1}{r^2}\int \frac{d(\cos kr)}{dk} \frac{f(k)\, k\, dk}{\sqrt{k^2-m^2}}$$

$$\psi(\bar{r},0) = -\frac{\cos kr}{r^2} \frac{f(k)\,k}{\sqrt{k^2-m^2}}\Bigg|_{m_T}^{\infty} + \frac{1}{r^2}\int_{m_T}^{\infty} \frac{d}{dk}\left(\frac{f(k)\,k}{\sqrt{k^2-m^2}}\right)\cos kr\, dk$$

If $f(k)$ goes to zero at least as fast as $\sqrt{k^2-m^2}$ does, then ψ decreases at least as fast as than r^{-2}. The last integral goes to zero for $r \to \infty$, by the Riemann-Lebesgue lemma ([69] Whittaker & Watson) if we assume $f(k)$ falls off sufficiently fast for large k to satisfy the conditions of the lemma.

If we integrate the second term by parts again we obtain:

$$\frac{1}{r^3} \sin kr \frac{d}{dk}\left(\frac{f(k)\,k}{\sqrt{k^2-m^2}}\right)\Bigg|_{m_r}^{\infty} - \frac{1}{r^3}\int \frac{d^2}{dk^2}\left(\frac{f(k)\,k}{\sqrt{k^2-m^2}}\right)\sin kr\, dk$$

We see that the first term vanishes if f(k) has a zero at least of the order 3/2 at k=m. The second term falls off faster than r^{-3} since the integral goes to 0 as $r \to \infty$ (the Riemann-Lebesgue lemma).

This procedure could be repeated indefinitely depending on how high the order of the zero at k=m is assumed to be. By suitably choosing f(k) we see that the associated ψ falls off for large r as rapidly as desired.

3. In Chapter III we found the condition for a sign change of the radial component of the acceleration of a tachyon produced by a central field of force. The relation of these considerations to Cherenkov radiation and the dynamics which accompany it is shown in this footnote. In 1963 G. M. Volkoff ([63] Volkoff), publicized the fact that the electric field within the Cherenkov cone of a particle exceeding the velocity of light in a nondispersive medium points towards the positively charged particle. Hence a static positive charge behind the Cherenkov cone would experience a force and acceleration towards the radiating positive particle. These two counter-intuitive direction reversals are seen to be

complementary and to occur for the same conditions on cos θ. Two positively charged particles, one moving faster than light, the other slower, begin to attract each other when the rest particle falls within the Cherenkov cone of the tachyon.

For the case of a particle emitting electromagnetic Cherenkov radiation in a vacuum, Sommerfeld in 1904 found a mysterious speeding up produced by the radiation reaction force.

4. Note that the reinterpretation principle doesn't change any of the physics of tachyons, it only changes the labels we attach to things. In particular, this principle cannot help us with the fact that the Lorentz covariant generator of time translations (usually called the energy) is unbounded below.

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